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"VALUE AT RISK BASED ARIMAGARCH MODELING ON TRADED SECURITIES: CASE STUDY OF THE ZIMBABWE STOCK EXCHANGE STOCKS (2010– 2016)."

By

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Approval form

The undersigned certify that they have read and recommend to the Bindura University of Science Education for acceptance of a dissertation entitled **"VALUE AT RISK BASED ARIMA-GARCH MODELING ON TRADED SECURITIES: CASE STUDY OF THE ZIMBABWE STOCK EXCHANGE STOCKS (2010– 2016)."**

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DEDICATION

I dedicate this dissertation to Katrina Dimingo, Babra Dimingo, Tatenda Ncube and Richard Dimingo who have made sacrifices towards my personal and professional endeavors. I thank you for believing in my dreams.

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ABSTRACT

Value at Risk using GARCH models remains the workhorse model in risk management as financial markets are becoming more complex followed by increased uncertainty worldwide. This research attempts to explore the comparative ability of the GARCH models in modeling and forecasting Value at Risk of the Zimbabwe Stock Exchange. The study used daily data of ZSE Industrial index for a period from 2010 to 2016. Data analysis was done using R version 3.5. The symmetric ARIMA-GARCH (1, 1) and asymmetrical ARIMA-EGARCH (1, 1) models were compared on the basis of AIC, BIC and log likehood values. The results shows that asymmetric ARIMA-EGARCH (1, 1) models outperforms the symmetric ARIMA-GARCH (1, 1). The Kupiec and the Christoffersen test were used for evaluation of VaR calculated using the different GARCH models. The most adequate GARCH family models for estimating VaR in the Zimbabwe Stock Exchange are the asymmetric ARIMA-EGARCH model with student-t distribution at 5% level of significance. Basing on the research findings a recommendation was also made to the investors and the stakeholders of the stock market to use an asymmetric GARCH model especially ARIMA-EGARCH model with student-t distribution that can capture leverage effect and risk factors in predicting VaR for the stock market. A recommendation was made for further studies that comparison of ARIMA-GARCH and other models like Multifactor Risk Approach, Risk metrics and Monte Carlo Simulation would help in the search of appropriate model for VaR estimation.

Keywords: Value at Risk, backtesting, GARCH models, ZSE

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ACRONYMS

ACF	**Autocorrelation Function**
AIC	**Akaike Information Criteria**
ARCH	**Auto Regressive Conditional Heteroscedasticity**
ARIMA	**Autoregressive Integrated Moving Average**
BCBS	**Basel Committee on Banking Supervision**
BIC	**Bayesian Information Criteria**
GARCH	**Generalized Auto Regressive Conditional Heteroscedasticity**
E-GARCH	**Exponential Generalized Auto Regressive Conditional Heteroscedasticity**
I-GARCH	**Integrated Generalized Auto Regressive Conditional Heteroscedasticity**
PACF	**Partial Autocorrelation Function**
Q-Q	**Quantile-Quantile**
VaR	**Value at Risk**

ZSE	**Zimbabwe Stock Exchange**

CHAPTER 1

1.0 INTRODUCTION

Accurate modelling of risk is of paramount importance in finance. Financial markets are becoming more global and complex due to their exposure to various types of risks. Among all kinds of risks, the most prominent of these are market risks. Market risks have become increasingly prevalent across the whole world including in developing markets like Zimbabwe. Market risk possess overwhelming potential of doing damage to many financial institutions, as a result the task for financial firms and stock markets to apply risk management tools is even more compelling. One of the widely used risk management tool is Value at Risk (VaR). According to analysts VaR is slowly replacing standard deviation or volatility as the most widely used measure of risk. Value-at-Risk is defined as the worst loss over a target horizon with a given level of confidence (Jorion, 2007). Almost every investor at some point in time needs to know the maximum possible loss in an investment. Value at risk as a tool for risk assessment provide an answer.

The key application of VaR is for assessing market risk. However, VaR is not a consistent method for measuring risk, as different VaR models will come up with different VaR results. The great availability of VaR technique has put researchers and risk measurement managers in difficult situation when using VaR since there are no single and standardized criteria to determine which method and model is the best. Hence evaluation towards performance of VaR methods and selection of appropriate VaR methods become very important. Due to the financial crisis a number of models have been developed that are specifically suited to estimate the conditional volatility and extreme market risks. The most well-known and frequently applied models for market risk are the econometric Generalized Autoregressive Heteroscedasticity (GARCH) models in VaR estimation.

1.1 BACKGROUND TO THE STUDY.

The growing interest of foreign financial investors to invest in emerging financial markets highlight the importance of accurate market risk quantification .Accurate risk quantification helps investors to better prepare themselves for uncertainties and risk. In empirical finance risk is broadly defined as volatility. The market risk situation of ZSE has been worsening over the years as weak economic activities have resulted in ZSE experiencing declining investor interest and trading volumes in the past five years. A significant number of counters have delisted from the local bourse after failing to meet the listing requirements, which is an indication that the volatile climate has dogged the bourse since dollarization in February 2009.

The ZSE once ranked among the best performing bourses in Africa, has gone from bad to worse as the economy has maintained a downward trend for more than a decade. Weak investor sentiment has haunted the local exchange the bourse losing more than 3.5 billion of its value since 2013 (Chronicles, September 19 2016).Currently several companies on the ZSE are heavily undercapitalized, a condition that has impeded performance for most listed companies. Raising fresh equity remains essential for ZSE listed companies at the height of the current liquidity constraints.

Weakening economic performance, political instability and the bond notes which has been put in circulation has affected the Stock market performance. What makes it difficult is that the local bourse remains fragile, thus has resulted in a crisis on the equity market where investor sentiment has been hit by low returns and declining corporate earnings. Since less than 10 companies are trading positively it means that investors are losing money as stock prices tumble due to the underperforming economy. There is need to revitalize investor confidence and one way of achieving this is having an accurate and correct prediction of market risk.

1.1.2 BRIEF BACKGROUND OF THE ZIMBABWE STOCK EXCHANGE.

The first stock market in Zimbabwe was during the Rhodesia era in June 1894.During that time three more exchange were established in Bulawayo, Gwelo(Gweru), and Umtali(Mutare).The British South African Company and other mining companies were the beneficiary of the exchanges Stock exchange is the official stock exchange of the Zimbabwe. It has been open to foreign investments since 1993. The performance of the Zimbabwe Stock Exchange is a mirror of what is happening in the economy. The stock market currently lists 65 equities. The Zimbabwe stock exchange has two indices namely the Industrial Index and the Mining index. The Zimbabwe Stock Exchange operates according to the Stock Exchange Act (chapter 198). The basic purpose was to bring transparency in the stock market. Listed companies must disclose information in time and in complete and accurate manner to the exchange and public on regular basis. Trading has been done manually on the Zimbabwe Stock Exchange and was conducted in a daily call over that begins at 10:00 and ends before noon.

1.2 STATEMENT OF THE PROBLEM

The search for appropriate measure to quantify market risk has become of enormous importance in financial markets. Market risk arise from negative price of financial assets due to unfavorable market conditions. Financial markets at the current stage of world economy are becoming more complex and global. It is transparent that increasing complexities bring increasing uncertainties and risks. As a result, investors and financial analysts are concerned about uncertainty of returns on their portfolio, caused by variability in the speculative market prices and the instability of business performance. Therefore the task for firms to correctly estimate risks for better preparation of uncertainties is even more compelling. Broad agreement now exist among regulatory bodies and security firms of the usefulness of VaR using GARCH models as a risk management tool. However, there is far less agreement on the issue of how it is calculated and among the GARCH models which of them is the best volatility model that can give an accurate measure of VaR In order to address the above problem ARIMA-GARCH with different error distribution and confidence intervals are used to see which of them accurately measures the Value at Risk.

1.3 AIM OF STUDY

This study attempts to close the gap in determining the best model of forecasting market risk to aid on efficient allocation of resources for investment. To this end, the study is to identify the best performing VaR model in the GARCH family for Zimbabwean stock market.

1.4 OBJECTIVES

The study sought to attain the following objectives:

1. To test the relative performance of the GARCH family models in estimating and forecasting VaR of the ZSE.
2. Evaluating the accuracy of the estimation of VaR calculated by the different GARCH models through backtesting.

1.5 RESEARCH QUESTIONS

The study seeks answers to the following specific research questions which reflect the research objectives:

1. Which GARCH type model provide the most accurate estimation of VaR?
2. How can the accuracy of different VaR based GARCH models be evaluated?

1.6 SIGNIFICANCE OF STUDY

- The government as the regulator of the ZSE will find this study valuable in respect of coming up with regulations that can ensure that the ZSE enhances its level of market efficiency. An assessment of the market risk of the stock market through the stock exchange will also enable policy makers to evaluate the effectiveness of the transmission of policy through the stock exchange market.
- The research is expected to benefit investors, portfolio managers and stock broking firms on the ZSE. When stock market risk increases, risk starts to averse investors, who in turn tend to reduce their holding of equities relative to safe assets such as Treasury bills. As a proxy of risk, Value at Risk is not only of great concern to investors but also to policy makers. On the other hand, policy makers are mainly focused on the effect of market losses on the stability of financial markets in particular and the whole economy.
- Finally, the research will contribute to the existing literature on evaluating different GARCH type models on quantifying market risk using VaR by examining its applicability

to developing countries like Zimbabwe. It will therefore help to contextualize the theories to the Zimbabwean situation. Hence academics will benefit from the research findings as it will be a source of reference for other students and staff of BUSE who might in future want to carry out research on Value at Risk.

1.7 JUSTIFICATION OF STUDY

A number of studies on Value at Risk have been conducted with various risk model specifications and different portfolio combinations. However, there are still plenty of areas to explore on this topic. Previous studies mainly focus on VaR using different parametric and non-parametric GARCH models. Moreover, the studies focus on the use of different error distribution on volatility models such as the normal distribution, student-t distribution and skewed-t distribution. Past studies have been also concerned with forecasting VaR using the symmetric and asymmetric ARCH/GARCH models mainly in developed markets. Nevertheless, few of them have applied VaR with ARCH/GARCH to emerging financial markets. As they remain forefront to the academic research this study also consider these models, but focusing on an emerging market such as Zimbabwe Stock Exchange. Therefore the limitations of previous studies combined with the author's personal interest have worked as prime motivation to further explore the topic of VaR using GARCH family models and different backtesting methods.

1.8 ASSUMPTIONS

- The GARCH models assume non-normal conditional variance that is clustering of periods of volatility (heteroscedasticity).
- Given that the sampled 61 industrial firms compose 94% of the ZSE listed firms, it is a sample to represent the stock exchange and the stock market as a whole.
- The data collected from the various sources is reliable.

1.9 LIMITATIONS

- Time is always a constraint and so with limited time the researcher did not manage to consider other forecasting methods of such as the Monte Carlo Simulation, Historical simulation and exponentially weighted moving average.
- There were also some missing values in the data as trading does not occur on weekends and public holidays. This could have affected the model.
- Empirical research have focused solely on the ZSE and therefore the obtained findings cannot be generalized to other emerging financial markets.

1.10 DELIMITATIONS

1.11 DEFINITION OF KEY TERMS

Value at risk- is the worst loss over a target horizon with a given level of confidence.

Market risks- is the possibility of an investor experiencing losses due to factors that affect the overall performance of the financial markets in which he or she is involved.

Trading securities- Trading securities is a category of securities that includes both debt and equity securities, and which an entity intends to sell in the short term for a profit that it expects to generate from increases in the price of the securities.

1.12 ORGANISATION OF STUDY

This study is divided into five chapters. Following this introductory chapter is chapter two which reviews relevant and related empirical literature on Value at Risk GARCH modelling of market risks. Chapter three gives a concise description of methods used to model value at risk as a way of tackling research objectives. It also presents an overview of the research design, as well as data collection and analysis procedures that were used. The fourth chapter furnishes a thorough presentation of the research findings and provides analysis of the data used in this chapter. Finally, chapter five concludes the study, makes recommendations and identifies areas for further study.

1.13 CHAPTER SUMMARY

This introductory chapter clearly highlighted the subject matter in this research, as well as point out the main focus of this study. This section of the research shows the background checks that motivated the researcher to engage in this research. The background, statement of the problem, significance, limitations, and delimitations were outlined in this chapter. The background of the chapter pointed out that there is need for a risk management tool to quantify and forecast risk in trading securities such as the ZSE. The statement of the problem defines the problem to be tackled and this is followed by research objectives that will provide solutions to the problem when they are realized. The chapter also indicated that the research will be of benefit to investors, the government, policy makers and regulators of ZSE.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

Value at risk is one of the most interesting topics in empirical finance that has been extensively studied by researchers in developed markets. Those studies have primarily looked at the application of different VaR models and applied different backtesting methods to evaluate the accuracy of the VAR models. Despite the extensive literature and empirical research of estimation of VaR models in the major developed financial markets, literature dealing with VaR calculation in emerging financial market is very scarce. The situation is even worse as far as the Zimbabwe stock exchange is concerned. Bucevska (2012) claims that the short historical time-series data did not allow for the performing of reliable econometric analysis since most of the emerging stock markets were established in the early 1990s. This research paper tries to extend the limited empirical research on VaR estimation and forecasting in emerging financial markets by testing the relative performance of a number of GARCH family models in the estimation of the Zimbabwe stock exchange. In this chapter the researcher cites or quotes various authorities that contributed information that was used to enlighten the researcher on the research topic. The information in the chapter also assisted the researcher in adopting the appropriate model for the study.

2.1 RELATED PAST RESEARCH

Batsirai Winmore Mazviona and Milton Webb Ndlovu (2015) studied the Day of the week effect of the Zimbabwe Stock Exchange using a non-linear GARCH model as well as the (Exponential General Autoregressive Conditional Heteroskedastic) E-GARCH model. Wellington Garikai Bongai (2014) wrote a research journal on the stock returns analysis and volatility of the Zimbabwean Stock Market. In his study he investigates the time series behavior of the stock returns for the Zimbabwe stock market. He tested for the asymmetric effect using the GARCH and ARCH model. He argued that the main purpose of forecasting volatility are the for the risk management, for asset allocation and taking beta on the future volatility. He also reviewed that there is a strong relationship between volatility and market performances.

For the similar studies that have been done in Zimbabwe concerning the volatility measures, Dennis Murekachiro made a study of time series volatility forecasting Literature Review 12 of the Zimbabwe Stock Exchange. In his study he used the univariate General Autoregressive Conditional Heteroscedasticity (GARCH) models using both symmetric and asymmetric models.

Sunde and Zivanomoyo (2007) in their study looked at the market behavior of the ZSE during the Zimbabwe dollar era from January 1998 to November 2006, whilst Mazviona and Nyangara (2013) concentrated on the market behavior during the United States dollar era from 19 February 2009 to 28 June 2012. The former used monthly data whereas the latter used daily data in their studies. However, both studies converged to the same conclusion that the ZSE is not a weak form market efficient, despite using different methodologies. In all essence this means that the ZSE has always been not a weak form efficient market, as it still maintains the same trends despite currency change.

2.2 THEORATICAL FRAMEWORK

The theoretical framework will firstly introduce and explain VaR, which is the nucleus of this entire research. Secondly parameters of VaR will be presented followed by a part covering error distributions, then followed VaR methodologies putting more emphasis on GARCH based models used in this research. Lastly, the tests used for evaluating the VaR forecasts will be presented and explained.

2.2.1 VALUE AT RISK

VaR has become a key component in the management of market risk for many financial institutions. In 1994, J.P Morgan introduced VaR a risk control methodology known as RiskmetricsTM. Over the last few years, it has become popular and is regarded as the masterpiece in financial risk management. Theoretical background for the VaR method is given by Jorion (1996), Duffle and Pan (1997) and Dowd (1998).

VaR took on greater importance when the Basel Committee recommended its use through the 1996 amendment to the 1988 Basel Accord (Basel Committee on Banking Supervision [BCBS].The BCBS also recommended VaR as the international standard for regulatory purposes. In 2006, the Basel Committee refined the regulation to the use of VaR allowing financial institutions to use their own internal VaR models subject to such models being recognized by the regulator. Moreover, the committee allowed banks to use internal market risk management models to fulfill their requirements regarding capital adequacy. VaR measures the risk of future losses from a specific financial assets for a certain holding period. In probabilistic terms, VaR is simply a quantile of the loss distribution (McNeil et al, 2002).

2.2.2 MATHEMATICAL DEFINITION OF VAR

Apart from theoretical definition it is important to have a concrete understanding of VaR at a mathematical point of view. Mathematically, VaR of a portfolio can be defined as the p quantile of the profit and loss among all quantiles in the return distribution (Jobayed, 2017). The formula is shown in equation (1)

$$\boldsymbol{VaR_t p = -F^{-1}(P|\Omega_t)}$$

In the equation above VaR_tp represents value at risk of a portfolio at probability (p) at time (t).$F^{-1}(p|\Omega_t)$ denotes the quantile function of P&L distribution and is time varying with respect to the change of portfolio composition Ω_t. The negative sign refers to the normalization and quotes VaR as losses. For example, if VaR_t (0.01) is found to be $20,000, it interprets that 1% of the time, portfolio may face a loss that is more than $20,000 (Campbell 2005:4)

2.2.3 ERROR DISTRIBUTIONS

For the models to fully function, the error term has to have zero mean that is $\varepsilon t \sim (0,1)$, where the error term in this case is normally distributed with zero mean and variance one. The general formula for density function of the Normal distribution of error terms is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\varepsilon_t-\mu)^2}{2\sigma^2}}$$

Where μ constitute the mean and σ is the standard deviation. The fatter tails, frequently observed in stock returns, are allowed for in the Student's t distribution assumed by Bollerslev (1987) which is given by the density function

$$f(x) = \frac{\Gamma\left(\frac{df+1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)\sqrt{\pi \cdot \beta \cdot df}}\left(1 + \frac{(x-\mu)}{\beta \cdot df}\right)^{-\frac{1+df}{2}}$$

Where Γ (·) is the gamma function, df is the degrees of freedom and an assumption that df> 2 is made. The t distribution converges to the Normal distribution as df → ∞.The *t* distribution has heavier tails which means that the probability of values falling far from its mean is higher than that of the normal and it is also symmetric and bell-shaped like the normal distribution(Supervisor, 2013).

2.4 VALUE AT RISK METHODOLOGIES

A variety of methods can be applied for calculation of VaR, but there are core models that are common in the literature and practice. These models can be described as core models because many models have been created based on one or more of these. There are three main methods to calculating value at risk which are Historical Simulation, Variance Covariance and Monte Carlo Simulation("Comparison of Value-at-Risk Estimates from GARCH Models," 2011). VaR methodologies are subject to criticism for many reasons, mostly because they do not consider time dependency in estimating volatility. Unconditional parametric methods assume that the returns are correlated and volatility is time invariant (Dowd, 2002). The historical simulation method makes an assumption that it is justified to make forecasts based on the past performance which can be misleading, particularly when structural breaks occur in volatility (Danielsson, 2011). All these models' drawbacks have made the GARCH models gain much popularity and, indeed, dozens of empirical studies show the reliability and usefulness of this methodology. An overview of the GARCH based models will be explained in this section, since they will be used as a benchmark in this research.

2.4.1 GARCH (p,q) model

GARCH (p, q) models are examples of symmetric models. Symmetric GARCH-type models consider negative and positive error terms to have symmetric effects on volatility in other words negative and positive shocks have the same effect on volatility.an example of such model is the GARCH (p, q) model. The GARCH model came about as an extension to ARCH model after Bollerslev noted various constraints of the ARCH model. The presented extension of the ARCH model is denoted as GARCH (p, q) where p represents the order of the GARCH elements and q represents the order of the ARCH elements. The GARCH (p, q) is given by

$$\sigma_t^2 = w + \sum_{i=1}^{q} a_i\, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-1}^2$$

q represents the order of ε_t^2 and p represents the order of σ^2. It is necessary to impose conditions, such as w>0, $a_i > 0$, $\beta_i > 0$ and $\sum_{i=1}^{q} a_i + \sum_{i=1}^{p} \beta i < 1$ to obtain a positive conditional variance and stationarity.

2.4.2 EXPONENTIAL- GARCH (EGARCH) model

The EGARCH model is an example of asymmetric GARCH model in which negative and positive shocks does not have the same effect on volatility. In 1991 Nelson introduced the exponential GARCH model. Nelson (1991) noted that his EGARCH model can overcome three main drawbacks of ordinary GARCH models. The three main drawbacks are negative correlation between current and future returns which is excluded by the GARCH model assumption, GARCH models imply parameter restrictions and GARCH models do not consider asymmetric variance effects EGARCH (*q, p*) is given by (Terasvirta, 2006):

$$\log \sigma_t^2 = w + \sum_{i=1}^{q} [\alpha_i \varepsilon_{t-i} + \lambda_i (|\varepsilon_{t-i}| - E|\varepsilon_{t-i}|)] + \sum_{i=1}^{p} \beta_i \log(\sigma_{t-i}^2)$$

The EGARCH model has a number of advantages over the GARCH *(p,q)* model. The most important one is its logarithmic specification, which allows for relaxation of the positive constraints among the parameters. Another advantage of the EGARCH model is that it incorporates the asymmetries in stock return volatilities. The parameters α and γ capture two important asymmetries in conditional variances. If γ < 0 negative shocks increase the volatility more than positive shocks of the same magnitude. Due to the parameter α expected to be positive, large shocks of any sign will be of larger impact compared to small shocks. Another advantage of the EGARCH model is that it successfully captures the persistence of volatility shocks(Bucevska, 2013).

2.5 MAXIMUM LIKELIHOOD GARCH ESTIMATION

After describing the properties of selected GARCH family models, there is need to know how the models are estimated when the only variable is the data on asset returns. The common methodology used for GARCH estimation is maximum likelihood assuming independent and identically distributed innovations. The parameters of the GARCH model can be found by maximizing the objective log likelihood function:

$$ln\ L(\theta) = -\frac{1}{2}\sum_{t=1}^{n}[\ln(2\pi + \ln(\sigma_t^2(\theta)) + z_t^2(\theta)]$$

where θ is the vector of parameters ($\mu, \omega, \alpha_i, \beta_j$) estimated that maximize the objective function $ln\ L(\theta)$; z_t represents the standardized residual calculated as $\frac{\Delta y_t - \mu}{\sqrt{\sigma_t^2}}$.The other symbols have the same meaning as above described. Maximum likelihood estimates of the parameters can be obtained via nonlinear least squares using Marquardt's algorithm.

2.6 BACKTESTING

Backtesting is an important part of the Value-at-Risk model evaluation process. It takes the values that have been calculated by the selected model and tests if the model has been accurate enough to justify its use on a given portfolio. The tests are often put in two sets of groups, unconditional coverage and independence. Unconditional coverage counts the frequencies of violations. A violation is when the actual return exceeds the Value-at-Risk number for that date. If the Value-at-Risk level were 5%, from a sample of 100 Value-at-Risk estimates against actual return observations, it would expect five of them to be violations. The test for independence makes assumption for the observations to be independent of each other, based on that, when a violation happens for two or more consecutive days there might be a problem with the model. Two backtest methods will be described in the following sections.

2.6.1 KUPIEC TEST

The Kupiec test is another backtesting method used to determine the consistency of violations with the given confidence level. In this test an assumption is made that the number of exceedances over time follows the binomial distribution. If the number of exceedances substantially differs from what is expected, then the risk model's adequacy is questionable (Danielsson, 2011). To perform the test number of actual violations (*E*), number of observations (*N*) and the VaR probability level (*p*) are needed. Assuming *E* is distributed by *Bin (N, p)* the null hypothesis is:

$H_0: p = p_0, \quad p_0 = 0.01$ or 0.05

And p is estimated by $\frac{E}{N}$. We test whether the observed number of violations *E* is considerably different from the expected number of violations $p*N$. As Kupiec (1998) proposed, this test is based on the likelihood ratio test.

$$LR = 2log\left[\frac{(1-\frac{E}{N})^{N-E}\frac{E}{N}^{E}}{(1-p_0)^{N-E}{p_0}^{E}}\right] \sim \chi^2(1)$$

Where

- *N* is the number of observations used to forecast VaR values
- *E* is the observed number of actual exceedances.

Despite that Kupiec's test does not assume the distribution of the returns, it still provides good results in the predictive accuracy of the model. On the other hand, the test has some restrictions in practice, namely the sample size needs to be large and it overlooks the independence of the actual violations (Dowd, 2002).

2.6.2 Christoffersen's conditional test

As noted above, Kupiec's test is a test of unconditional coverage as it does not consider independence of the violations or exceedances. To check the volatility cluster in a risk model, in

1998, Christoffersen proposed a test based on the conditional coverage. This Christoffersen's test determines whether a violation occurred on a particular day conditionally depends on the previous day's result (Christoffersen, 1998). The procedure of carrying out the test is given as in Danielsson (2011).At first we need to compute the following probabilities:

$P_{ij} = Pr\ (m_t = i|m_{t-1} = j)$ where *i and j* are either 0 or 1 and m_t means whether a VaR exceedance occurs at time *t*.The first-order transition matrix is given by:

$$\Pi_1 = \begin{pmatrix} 1 - p_{01} & p_{01} \\ 1 - p_{11} & p_{11} \end{pmatrix},$$ 0 = no exceedance

1 = exceedance

Where

-$p_{01}$is the probability of a violation or exceedance when there was no violation on a previous day

- p_{11} is the probability of two consecutive violations

Under the null hypothesis H_0, we assume that there is no volatility cluster which means all the violations occurred are independent that is p_{01}= p_{11} = p and the transition matrix is:

$$\Pi_0 = \begin{pmatrix} 1 - \hat{p} & \hat{p} \\ 1 - \hat{p} & \hat{p} \end{pmatrix}$$

$$\hat{p} = \frac{v_{01} - v_{11}}{v_{00} + v_{10} + v_{01} + v_{11}}$$

2.7 EMPIRICAL FINDINGS FROM OTHER RESEARCHERS

Zikovic and Aktan (2009) investigated the relative performance of a variety of VaR models with the daily returns of Turkish(XU100) and Croatian (Crobex) stock index prior to and during the global 2008 financial crisis. Zikovic and Filler (2009) test the relative performance of VAR and ES models using daily returns for sixteen stock market indices. The market indices they studied eight were from developed and eight from emerging markets. The study was conducted prior to and during the 2008 financial crisis. However, the main limitation of their studies is the fact that they have tested the relative performances of VaR models at the very beginning of the global financial crisis. Due to the short sample period it is recommended that future research with the inclusion of a longer period is needed in order to obtain estimates with greater precision.

Nyssanov (2013) conducted a study on the performance of classical approaches and GARCH family models. The models were evaluated and compared in estimation one-step-ahead VaR. The classical VaR methodology used included historical simulation (HS), Risk Metrics, and unconditional approaches. The classical VaR methods, the four univariate and two multivariate GARCH models with the Student's t and the normal error distributions were applied to 5 stock indices and 4 portfolios to determine the best VaR method. Four evaluation tests were used to assess the quality of VaR forecasts: Violation ratio, Kupiec's test, Christoffersen's test, Joint test. The results point out that GARCH-based models produce far more accurate forecasts for both individual and portfolio VaR. Risk Metrics gives reliable VaR predictions but it is still substantially inferior to GARCH models.

In the study of Bucevska (2013) daily returns of the Macedonian stock exchange index-MBI 10 were used in testing the performance of the symmetric GARCH (1,1) and the GARCH-M model as well as of the asymmetric EGARCH (1,1) model, the GARCH-GJR model and the APARCH(1,1) model with different residual distributions. The most adequate GARCH family models for estimating volatility in the Macedonian stock market were found to be asymmetric EGARCH model with Student's t-distribution, the EGARCH model with normal distribution and the GARCH-GJR model. A conclusion was made that the obtained findings bear important implications regarding VaR estimation in turbulent times that have to be addressed by investors in emerging capital markets. However, the main limitation of the study was that the research dealt on symmetric GARCH only and a recommendation was made that in future inclusion of asymmetric GARCH-type models and testing and comparing their predictive performance would be better.

In the study of Berggren and Folkelid (2014) of identifying the best volatility model for Value-at-Risk (VaR) estimations. They estimated 1 % and 5 % VaR figures for Nordic indices and stocks by using two symmetrical and two asymmetrical GARCH models under different error distributions. Out-of-sample volatility forecasts were produced using a 500 day rolling window estimation on data covering January 2007 to December 2014. The VaR estimates were thereafter evaluated through Kupiec's test and Christoffersen's test in order to find the best model. The results suggested that asymmetrical models perform better than symmetrical models albeit the simple ARCH is often good enough for 1 % VaR estimates(Berggren & Folkelid, 2014).

Notably, Brooks and Persand (2003) asserted that asymmetry is an important issue in the VaR framework, and therefore must be modeled in the volatility specification. To this end, Angelidis et al. (2004) evaluate the performance of an extensive family of ARCH models with three distributional assumptions (normal, student-t and GED) in modeling the daily Value-at-Risk of five stock indices. Based on the proposed quantile loss function, there was strong evidence that the combination of the student t-distribution with EGARCH models produces the most adequate VaR forecasts for the majority of stock market data.

Another study which investigated the daily Value-at-Risk (VaR) for 0050-ETFs returns of the Taiwan Stock Exchange from 2003 to 2007. The essential source of performance improvements between distributional assumption and volatility specification was identified utilizing symmetric (GARCH) and asymmetric (GJR-GARCH) volatility models under alternative distributions through two-stage models selection criterion. Empirical results indicated that the roles of distributional assumption and asymmetric volatility specification achieved their superiority at different confidence levels. Eventually, they encouraged that GJR-t/GARCH-HT model is a useful technique for conservative/aggressive risk managers against market uncertainty in volatile ETFs markets(Liu, Cheng, & Tzou, 2009).

The research carried by Mimetic (2015) implemented GARCH models that involve time varying volatility and heavy tails to the empirical distribution of returns, in selected Central and Eastern European emerging capital markets (Croatia, Czech, Hungary, Romania and Serbia). They showed that GARCH models with a t-distribution of residuals in most analyzed cases give a better VaR estimation than GARCH models with normal errors in the case of a 99% confidence level, while the opposite is true in the case of a 95% confidence level. The backtesting results for the crisis period showed that GARCH models with a t-distribution of residuals provide better VaR estimates when compared with GARCH models with a normal distribution, historical simulations or RiskMetrics methods.

So and Yu (2006) studied GARCH models, including RiskMetrics and two long memory GARCH (IGARCH, FIGARCH) models, in Value-at-Risk estimation, and found evidence that t-error models are superior to normal-error models in determining an appropriate value of VaR for long position at the 99% confidence level. Hung et al. (2008) found evidence that the proposed GARCH-HT model-based VaR approach achieves good accuracy and efficiency at both low and high confidence levels for alternative energy commodities when asset returns exhibit leptokurtic and fat-tailed features. On the other hand, Bams et al. (2005) reached similar conclusions, arguing that the GARCH (1,1)-t model is adequate for correctly assessing extreme losses for exchange rate positions.

Alou and Ben Hamida explored the relevance of asymmetry, long memory and fat tails in modeling and forecasting the conditional volatility and market risk for the Gulf Cooperation Council (GCC) stock markets. Various linear and non-linear long-memory GARCH-class models under three density functions were used to investigate this relevancy. Their results revealed that non-linear GARCH-class models accommodating long memory and asymmetry can better capture the volatility of returns. Interestingly, the FIAPARCH volatility model with skewed Student distribution was found to be the best suited for estimating the value at risk and expected shortfall for short and long trading positions. The model outperformed the other competing long-memory GARCH-class models and simple GARCH and EGARCH models. They concluded that long-memory, asymmetry, persistence and fat tails are important empirical facts in the GCC markets that should be taken into account when modeling and predicting volatility and assessing total risk (Alou & Ben Hamida, 2015).

Another empirical research was conducted which considers the adequacy of generalized autoregressive conditional heteroscedasticity (GARCH) model use in measuring risk in the Montenegrin emerging market before and during the global financial crisis. The main aim of the study was to investigate whether GARCH models are accurate in the evaluation of value at risk (VaR) in the Montenegrin stock market. The daily return of the Montenegrin stock market index MONEX was analyzed and evaluated using different backtesting methods. The backtesting results showed that none of the eight models used passed the Kupiec's test with 95% of confidence level, while only the ARMA (1,2)–N GARCH model did not pass the Kupiec test with a confidence level of 99%. The results of the Christoffersen's test revealed three models (ARMA(1,2)–TS GARCH(1,1) with a Student-t distribution of residuals, the ARMA(1,2)–T GARCH(1,1) model with a Student-t distribution of residuals, and ARMA(1,2)–EGARCH(1,1) with a reparametrized unbounded Johnson distribution which passed the joint Christoffersen test with a 95%confidence level. Finally, none of the analyzed models passed the Pearson's Q test, whether with 90%, 95% or 99% (Smolović, Lipovina-Božović, & Vujošević, 2017).

Alou and Hamilda (2015) explored the relevance of asymmetry, long memory and fat tails in modeling and forecasting the conditional volatility and market risk for the Gulf Cooperation Council (GCC) stock markets. Various linear and non-linear long-memory GARCH-class models under three density functions were used to investigate this relevancy. The results revealed that non-linear GARCH-class models accommodating long memory and asymmetry can better capture the volatility of returns. The FIAPARCH volatility model with skewed Student distribution was found to be the best suited for estimating the value at risk and expected shortfall for short and long trading positions. The model outperformed other competing long-memory GARCH-class models and simple GARCH and EGARCH models.

In the study of K. Makiel (2012) to find out whether VaR estimation is influenced by conditional distribution of return rates (normal, t-student, GED) and choosing the model which best estimates VaR on a selected example. Logarithmic return rates for the WIG-20 index from 1999-2011 were used to estimate various types of ARIMA-GARCH (1, 1) models. Applying relevant models VaR was calculated for the long and short position. The differences between the models were settled on the basis of the Kupiec test. The results have showed that there are significant differences between models with different conditional distributions. The VaR results were not only affected by the conditional distribution of the models but also by the type of the analyzed position. A conclusion was made that when measuring risk using Value at Risk by GARCH models, it is much more effective to calculate values based on the models other than normal conditional distribution.

The study conducted by Anton Ringqvist (2014) investigated several models that estimate the financial risk measure Value at Risk (VaR) with the objective to find the best model for the Swedish stock market. Using 1-day forecasted VaR at 95% and 99% level the following VaR models were compared Basic Historical Simulation (HS), age weighted HS (AWHS) and volatility weighted HS (VWHS) using a GARCH model, Normal VaR and t-distributed VaR. The study was performed on the Swedish stock exchange data OMXS and on the single stock series Boliden for the years 2005-2013. After running a backtest of the models it was found that the VWHS, where the volatility was modelled with a GARCH (1,1) model, estimates 1-day 95% and 99% VaR most accurately on the Swedish stock market and is therefore preferred to the other models. However the study was limited on VaR backtesting with a smaller out out-of-sample. The study recommended that since the model fitting of the 99% VaR was problematic due to the lower statistical power of the tests it would be interesting for future studies to run VaR backtesting with a bigger out-of-sample, i.e. bigger than four years of data,

2.8 SUMMARY

This chapter has presented the previous work done on Value at Risk modelling in using GARCH models and also methods used for evaluation. The chapter laid theoretical background to the study. Credit is to be given to the theoretical contribution of Jorion and Dowd who laid the foundations of VaR which have been of great relevance in this research. Results from literature show that GARCH-type models can be used to model VaR especially in trading securities. Empirical evidence reveals that GARCH modelling has become a popular method used by researchers in risk quantification through estimating and forecasting of VaR The chapter aided the researcher in coming up with some missing pieces in literature that need for execution to guarantee adequate solutions to a better risk management framework. The VaR vary with different error distributions, confidence intervals and different GARCH family models. Thus, Bucevska (2013 recommends comparison between symmetric and asymmetric GARCH-type models. Moreover, K. Makiel (2012) recommends the use other error distributions other than normal distribution. The following chapter will cover the research methodology that is the research design, data collection procedure in pursuit of generate solutions gap revealed in this chapter.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

In this chapter, methodology to be used to undertake empirical research that examines the research questions as they relate to Value at Risk GARCH modelling in traded securities. The chapter focuses on how data will be gathered, statistical methods and techniques adopted in order to meet the objectives pointed out in the first chapter of this research. It covers research design, research methodology, population and sampling, data collection and data presentation and analysis.

3.2 RESEARCH DESIGN

According to Selltiz as cited by Kumar (2011), research design is the arrangement of conditions for collection and analysis of data in a manner that aims to combine relevance to the research purpose with economy in procedure. A research design is a blueprint of the collection, measurement and analysis of data and gives guideline to the researcher on how to get relevant information. Borg and Ball (2003), define research design as an overall strategy that one choses to interrogate different components of the studying a coherent and logical way ensuring that one will effectively address the research problem. According to Leedy (2001) research design is the plan or structure of investigation used to detain evidence used to answer research problem. A research design can also be viewed as way of unearthing answers. In this research the researcher will use a descriptive research design.

3.2.1 DESCRIPTIVE RESEARCH DESIGN

The researcher adopted descriptive research design for this study since the method had several advantages which aid in the solutions to the research questions. According to Bryan & Bell, (2005) research design is the framework for the collection of data and the following analysis. Descriptive research design involves the process of collecting data in order to answer the questions concerning the current status of the objectives of the study. This research design is appropriate for this study because it ensures in-depth analysis and description of the various phenomena under investigation. The descriptive research design adopted in this research is based upon the assumption that the case of Zimbabwean Stock Exchange at period 2010 to 2016 is a typical of cases of various trading securities and therefore can provide an insight of which GARCH model the best is when forecasting VaR. Zimbabwe Stock Exchange will be used as a case study in this research.

3.4 DATA COLLECTION

Data collection is a systematic approach of gathering and measuring information. Data collection allows researcher to collect information that is needed to collect about study objects. It also enables the researcher to answer relevant questions and evaluate outcomes. The research is based on analysis of secondary methods of collecting data. Secondary data refers to relevant information already in existence prior to the carrying out of the research (Aaltio & Pia, 2009). Thus research is historical in nature as it is based on historical data. The data used in this research largely came from the Zimbabwe Stock Exchange official website, (www.zse.co.za). Seven-year data was gathered from the bourse from a period of 2010 to 2016 which is the post-dollarization of the economy in Zimbabwe. The data used consists of daily log returns on ZSE return index generating for the study period 1754 observations. The inspiration behind choosing daily data is that the stock markets react very quickly to new information.

3.5 RESEARCH TARGET

The research methodology lends itself to quantitative analysis concentrating on the companies listed on the Zimbabwe Stock Exchange. For the purposes of this research the Zimbabwean industrial index will be used because it is regarded as the benchmark index as it constitutes 94% of the total number of listed firms. The industrial index is used in this research, following Darskuviene (2010), who recommended the use of a large number of stocks. Indices that are made of numerous stocks tend to be a good representation of the whole market unlike those with small number of stocks. The sample size used 61 of the 65 listed industrial companies that trade on the

Zimbabwean bourse. The industrial index is made up of 61 listed companies in 17 sectors of the economy. The researcher used a sample size made up of all population firms which make up the industrial index.

3.6 RESEARCH INSTRUMENT

Kumar (2011) defines a research instrument as anything that becomes a means of collecting information for your study for example observation forms, interview schedules, questionnaires and software packages in quantitative analysis. R software package version 3.5 will be used for the data analysis in this research. The researcher used internet and textbooks to provide the theories, literature review and basic information. Microsoft excel shall be used to create a dataset of the daily stock index. In this study, daily returns Rt were calculated as the continuously compounded returns.

$$R_t = \ln \frac{P_t}{P_{t-1}}$$

Where R_t is the return on industrial price index and P_t and P_{t-1} is the price of the index at time t and t-1 respectively. Logarithmic returns in this study will be used as highlighted by Strong (1992, p. 533) that they are analytically more tractable when linking together sub-period returns to form returns over long intervals and are more likely to be normally distributed and so conform to the assumptions of the standard statistical techniques.

3.7 DATA PRESENTATION AND ANALYSIS

Data analysis process involves organizing the data and general categories, themes and patterns (Marshall and Rossman, 1995). According to Rossman (1995) data analysis is the process of bringing order, structure and meaning to the collected material from research collection phase. Data presentation means displaying analyzed statistical information for easier comprehension of trends. The deductive and inferential statistics were presented in tables, graphs, and some narrative aspect. The researcher will reflect on the data and give it meaning through descriptive and inferential statistics. This will enable the researcher to settle reasonable conclusions on which GARCH model is the best when modelling Value at Risk. The following preliminary tests were carried out to ensure credibility and exactness of results:

- Identify if data is Stationary
- Finding if data is Auto correlated
- Test for Normality

3.7.1 STATIONARITY TEST

According to Brooks (2008), all the variables in time series need to be tested for non-stationarity using unit toot test. To investigate whether the daily price index and its returns are stationary series, Augmented Dickey Fuller (ADF) test was employed. The stationarity or non-stationarity of a series can heavily influence its behavior and properties.

Augmented Dickey Fuller Test

It tests the null hypothesis that a unit root present in a time series sample. The Augmented Dickey Fuller statistics used is a negative number. The more the negative it is, the stronger the rejection of the hypothesis that there is a unit root at some level of confidence. The Augmented Dickey Fuller test is conducted by adding the lagged values of the dependent variable. In this research, the industrial index daily prices shall be first differenced to ensure stationarity in data.

3.7.2 AUTOCORRELATION TEST

Another feature of the time series data is autocorrelation. To get meaningful results, the covariance and correlation between difference disturbances must be zero. This requires that all disturbances are independently and identically distributed or not correlated with one another. Autocorrelation is often caused by either omitted variables or misspecification of the model. Serial correlation which is the presence of autocorrelation in the time series results in overestimated results. To evaluate the autocorrelation of the returns and squared returns the researcher used Ljung-Box test.

Ljung-Box test

The null hypothesis in the Ljung-Box test is $H_0 : \gamma_k = 0$ for the lags k = 1,…..K against

H_1: ∃k such that $\gamma_k \neq 0$.

The test statistic in the Ljung-Box test is given by:

$$Q(K) = T(T+2)\sum_{k=1}^{K}\frac{\gamma_k^2}{T-l} \sim H_0\chi^2(K)$$

The researcher shall use 12 lags in calculating the Q-statistic in R software to see if the return series is autocorrelated. The Q-statistic will then be compared with the chi-square critical value with 12 degrees o freedom.

3.8 MODEL SELECTION

Most commonly used methods for model selection are Akaike Information Criteria (AIC) and Bayesian Information criterion (BIC) values for two/more models, the lowest AIC or BIC value should be chosen. In this research the BIC shall be used. The maximum likelihood estimation procedures in the formulation of the test statistic from a log likehood functions for BIC is

$BIC_t = -2 \ln(L) + 2 * (N)$

Where k=model degrees of freedom N=number of observations. Since the GARCH models used in this research has a ARIMA component, the model with the best ARIMA order is selected on the basis of BIC.

3.9 MODEL SPECIFICATION

The study is going to compare two GARCH type models which are the GARCH (1, 1) and the E-GARCH (1, 1) based on normal and Student t distribution both with ARIMA (3.0.2) component. Two classes of models are going to be compared which are symmetric and asymmetric GARCH models. To evaluate the effectiveness of the VaR forecasts obtained two backtesting methods are going to be used which are the Kupiec and Christoffersen tests methodologies at 5% and 1% level of significance.

3.9.1 SYMMETRIC GARCH

GARCH (1,1)

For simplicity and reliability a GARCH (1,1) model shall be used which is given by

$$\sigma_t^2 = w + a_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

Where

- εt are returns with zero mean and unit variance
- w, α_1, β_1 are model coefficients

- $w > 0$, $\alpha_1 > 0$, $\beta_1 > 0$ and $\alpha_1 + \beta_1 < 1$
- $\sigma^2 = \frac{w}{1-\alpha_1-\beta_1}$, σ^2 is the unconditional variance of εt

$$E(\sigma_t^2 = E\left(E\left(\varepsilon_t^2 | \varepsilon_{t-j}, j = 1,2 \ldots\right)\right) = \sigma^2$$

3.9.2 ASYMMETRIC GARCH

The symmetric GARCH models described above cannot account for the leverage effects observed in stock returns. The tendency for volatility to decline when returns rise and to rise when returns fall is called leverage effect. The main drawback of symmetric GARCH models is that they are not able to respond asymmetrically to the rise and fall of returns. This research uses the E-GARCH (1.1) to capture the asymmetry and leverage effect of industrial stock returns.

E-GARCH (1,1)

The EGARCH (1,1) model is given by :

$$\log \sigma_t^2 = w + (\alpha\varepsilon_{t-1} + \lambda(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|))) + \beta \log(\sigma_{t-1}^2$$

- εt are returns with zero mean and unit variance
- $w, \alpha_1, \beta_1, \lambda$ are model coefficients
- α_{t-1} is a sign of asymmetry effect
- $\lambda|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|))$ is a magnitude effect

The presence of leverage effects is due to that (gamma) $\gamma < 0$. The impact is asymmetric if $\gamma \neq 0$

3.10 BACKTESTING

In this research two backtesting methods are going to be carried out which are the Kupiec and the Christoffersen test. The last 1200 observations are going to be used for backtesting as an estimation window. Usually, a section of the sample is used for estimation (estimation window), and the forecasts are tested on the rest of the observations. If the observed loss on a given day is greater than the one predicted by the model, the violation is recorded. Depending on the confidence level, violations are expected in 5% and 1% of observations for 95% and 99% VaR respectively. At the end of the backtest, the number of observed violations is compared to the number of predicted violations. The study is going to calculate violation ratio for the model as follows:

$$Violation\ Ratio = \frac{Number\ of\ observed\ violations}{Number\ of\ predicted\ violations}$$

The ratio should be as close to one as possible, values above one means that the model underestimates risk as more violations than predicted turn up during the backtesting. On the other hand, values below one are a sign of over-conservative model (overestimated risk) as there are less violations than predicted.

3.11 SUMMARY

This chapter provided the methodology that will be used during the research. The area of study, research design, data collection techniques and data presentation and analysis procedure were highlighted. On data analysis the test necessary to be carried out before the data is used for model fitting were discussed. Moreover the model selection and the GARCH models to be fitted were explained in terms of their general fitted model equations. Finally, the evaluation of which VaR model is the best through backtesting was explained. The chapter was laid in relation to recommendations made by the previous literature reviewed in the second chapter. The subsequent chapter focuses on data representation, analysis and discussion.

CHAPTER 4

DATA PRESENTATION AND ANALYSIS

4.0 INTRODUCTION

The goal of this research is to compare the relative GARCH type models in finding the Value at Risk of Zimbabwe Stock Exchange. The construction of the GARCH models will be discussed in this chapter. The analysis and results are based on financial data from the population sample and secondary sources like the Zimbabwe stock exchange website. Stationarity, auto correlation and normality have been tested and corrected on the data before fitting it into the model. This chapter gives an in-depth analysis of data where quantitative analysis techniques are employed. Data presentation is done using tables and graphs to give a vivid impression and facilitate easy of analysis.

4.1 TESTING FOR STATIONARITY

The unit root is carried out to test the null hypothesis that the return time series under observation is non-stationary or have a unit root. The Augmented Dickey Fuller Test was applied to check the stationarity of the industrial series. The Augmented Dickey Fuller test showed stationarity in the industrial return series with the lowest test-statistic of -23.00 as compared to -3.433,-2.863,-2.568 at 1%,5% and 10% level of significance. The conclusion drawn from the results is to reject the stated null hypothesis and conclude that the return series is stationary.

ADF Test Statistic	-23.0083	Critical value at 1%	-3.4396
		Critical value at 5%	-2.8648
		Critical value at 10%	-2.5685

Table 4.1: Summary of ADF Unit Root Test

4.2 TESTING FOR AUTOCORRELATION

Testing for autocorrelation is one of the preliminary test before modelling and forecasting time series data as identified in the methodology of this research. The ACF and PACF plots below indicates that the series is stationery as indicated by the ADF test. From the plots below the spikes shows the presence of serial correlation for the industrial index log return. The spikes of autocorrelation function (ACF) and partial autocorrelation function (PACF) are beyond limits implying the squared errors are autocorrelated with squared errors of past period. The Ljung-Box Q-statistic null hypothesis that all there is no autocorrelation will be rejected since the computed Q-statistic value of 406.20 is greater than Chi-squared critical value of 26.217 at 1% significance level after employing 12 lags. These results show that there is serial correlation in the log returns of ZSE. Moreover, rejecting the null hypothesis confirms presence of ARCH effects in the series. Presence of ARCH effects means that the econometric GARCH models can be used in forecasting VaR.

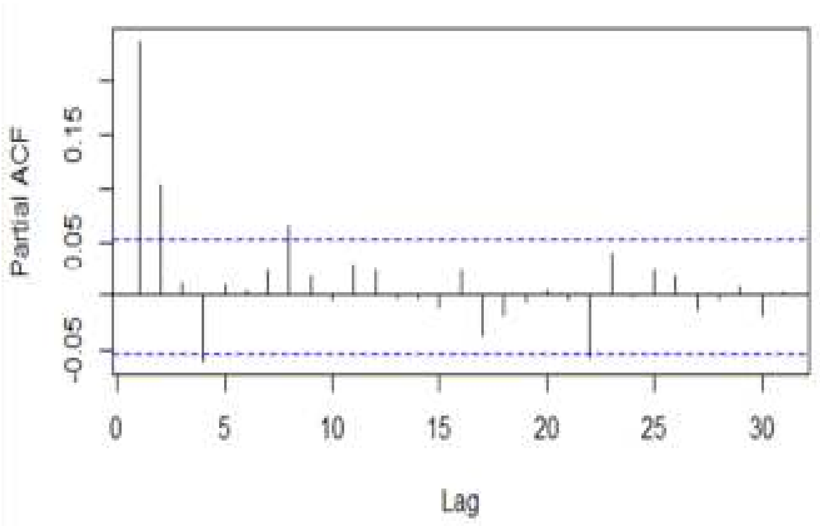

Figure 4.1: PACF plot of the ZSE Industrial index

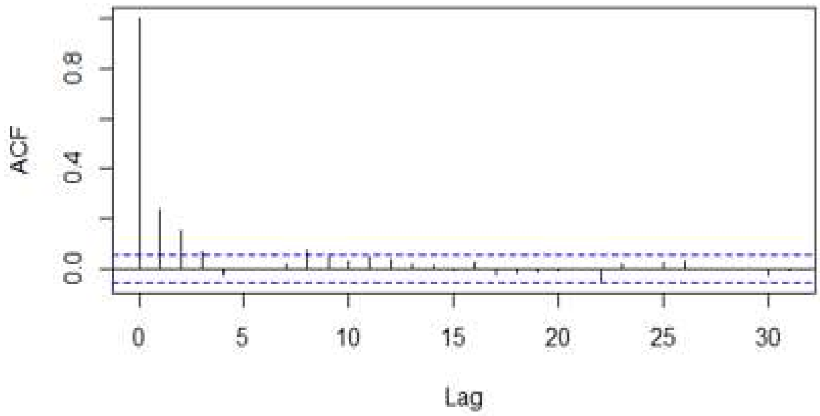

Figure 4.2: ACF plot of the ZSE Industrial index

4.3 NORMALITY TEST

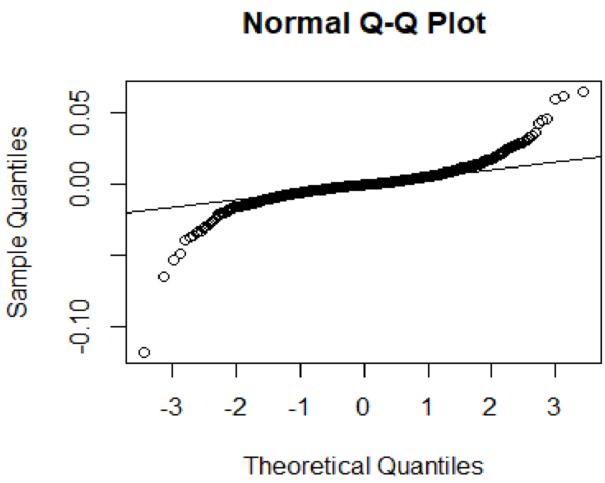

Figure 4.3: Q-Q Plot of Daily Stock Returns of the Industrial index

The Normal Q-Q plot of industrial index log returns has points which fall along a line in the middle of the graph, but curve off in the extremities. This shows that the returns exhibit more extreme values than would be expected if they truly came from a Normal distribution. The empirical distribution of returns deviates from normal. The above distribution in the plot allows the use of econometric approach in VaR estimation.

4.4 PRESENTATION AND ANALYSIS OF RESULTS

4.4.1 INFORMATION CRITERIA

Based on the Bayesian Information Criterion (BIC) in figure 4.4, it is estimated that the best model for modelling the logarithmic return series of Industrial index of ZSE is the ARIMA (3.0.2).Using the best model selected by the Information criteria also acts a way of removing autocorrelation in log returns. From the R output it can be noted the best model is the one with least value of BIC value. The ARIMA (3.0.2) has a least BIC value of -11946.03 which is the minimum Bayesian information criterion value compared to all other values in the output.

```
ARIMA(3,0,1) with non-zero mean : -11452.52
ARIMA(3,0,3) with non-zero mean : -11443.69
ARIMA(2,0,1) with non-zero mean : -11434.15
ARIMA(4,0,3) with non-zero mean : -11445.15
ARIMA(3,0,2) with zero mean     : -11469.03
ARIMA(2,0,2) with zero mean     : -11451.83
ARIMA(4,0,2) with zero mean     : -11463.56
ARIMA(3,0,1) with zero mean     : -11459.94
ARIMA(3,0,3) with zero mean     : -11468.65
ARIMA(2,0,1) with zero mean     : -11441.62
ARIMA(4,0,3) with zero mean     : -11452.55

Now re-fitting the best model(s) without approximations...

ARIMA(3,0,2) with zero mean     : -11415.85

Best model: ARIMA(3,0,2) with zero mean
```

Figure 4.4: BIC Information Criterion

4.5 GARCH MODELS FITTING

The parameters of the estimated models and goodness of fit for the models are given in Table 4.2.From the Ljung Box test for white noise behavior in residuals shown in Appendix section for different models used the residuals have p-values>0.05 and we fail to reject the null hypothesis, there is no evidence of autocorrelation in the residuals of all models used in this research. Hence the residuals behave as white noise. Looking at the standardized squared residuals and ARCH LM Tests, the p-values>0.05 and we fail to reject the null hypothesis hence there is no evidence of serial correlation in squared residuals for all ARIMA-GARCH models with different error distributions.

Looking at the output for the goodness of fit test, since the p-values>0.05, the normal distribution assumption is strongly rejected for all GARCH models fitted under normal distribution. When the p-value of normal distribution are greater than 0.05 we reject the null hypothesis and conclude that the model is not a good fit. On the other hand, the p-values of goodness of fit for the models under student-t distribution shows that the models are appropriate. The coefficient in the EGARCH models is significantly different from zero, which indicates presence of asymmetry. Looking at the parameter of ARIMA (3.0.2)-EGARCH (1.1) with normal distribution alpha, since alpha1 < 0, the leverage effect is significant and hence the volatility reacts more heavily to negative shocks. Moreover, the positive gamma parameter of ARIMA (3.0.2)-EGARCH (1.1) with Student-t distribution shows presence of asymmetry and that volatility reacts more heavily to positive shocks.

ESTIMATED ARIMA-GARCH MODELS

	ARIMA(3.0.2)-GARCH(1.1) normal distribution	ARIMA(3.0.2)-EGARCH(1.1) normal distribution
W	-0.000144	-0.000920
α_1	0.000010	-0.063350
β_1	0.501114	0.793008
gamma	-	0.673656
AIC	-6.9197	-6.9198
BIC	-6.8916	-6.8886
Log likelihood	6077.586	6078.631
	ARIMA(3.0.2)-GARCH(1.1) Student-t distribution	ARIMA(3.0.2)-EGARCH(1.1) Student-t distribution
W	-0.000288	-0.000286
α_1	0.516901	0.017494

β_1	0.305341	0.673297
gamma	-	0.604101
AIC	-7.2537	-7.2587
BIC	-7.2225	-7.2244
D.O.F	3	3
Log likelihood	6371.477	6376.885

Table 4.2: Estimated ARIMA-GARCH models.

Based on the minimum AIC and BIC values in table 4.2, the best fit model for the series are the ARIMA (3.0.2)-EGARCH (1.1). The ARIMA (3.0.2)-EGARCH (1.1) with error distribution student-t has the least Akaike Information criterion (AIC) of -7.2587 compared to -7.2537 of the ARIMA(3.0.2)-GARCH(1.1) . On the other hand, the ARIMA (3.0.2)-EGARCH (1.1) with normal has the least Akaike Information criterion (AIC) of -6.9198 compared to -6.9197 of the ARIMA(3.0.2)-GARCH(1.1) under normal distribution.

From the AIC and the BIC values it can be seen that the asymmetric E-GARCH models have minimum values than the symmetric GARCH model irrespective of the error distributions assume. Moreover, the log likelihood values supports that the ARIMA (3.0.2)-EGARCH (1.1) are the best model since they have the greatest log likelihood value. However, the log likelihood values are all relatively similar which means that only temporary inferences can be made therefore we need to analyze the backtest of these models before we can make any statements about the accuracy of performance of the models.

4.6 VaR BACKTESTING

The outcome of backtest results shows the number of exceedances for each VaR model, with an alpha level set at both one percent and five percent for the backtest .To illustrate the meaning of table 4.3 for ARIMA (3.0.2)-GARCH (1.1) under normal distribution figure 4.5 which is the 1% graph of the model is going to be used. The blue line represents the value-at-risk level, forecasted for a period length of 1200 observations with a one day moving window that refits every 1th step. All the returns are plotted as observed, some observations have returns lower than the value-at-risk level. These observations are called exceedances and are marked red in the graph.

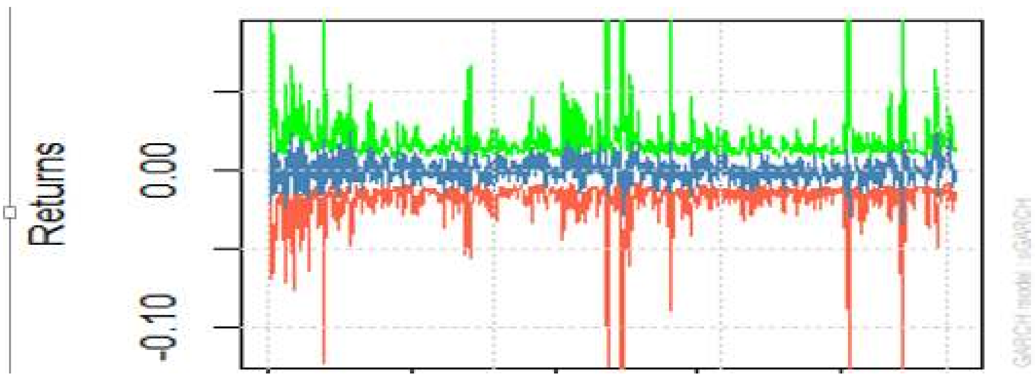

Figure 4.5: 1% VaR of ARIMA (3.0.2)-GARCH (1.1) under normal distribution.

Under normal conditions the blue line should touch the red line 12 times but in actual fact the graphs touch 10 times at 1% resulting in a failure rate of 1.3% as illustrated in table 4.3.

4.7 COMPARING BACKTEST RESULTS

	ARIMA(3.0.2)-GARCH(1.1)				**ARIMA(3.0.2)-EGARCH(1.1)**			
	Normal		**Student-t**		**Normal**		**Student-t**	
Backtest Length	1200							
VaR level	5%	1%	5%	1%	5%	1%	5%	1%
Expected Exceed	60	12	60	12	60	12	60	12
Actual VaR Exceed	44	10	47	10	46	16	59	9
Failure rates %	3.97	1.3	3.9	0.8	3.8	1.3	4.9	0.8
Violation ratio	0.73	0.83	0.78	0.83	0.77	1.33	0.98	0.75
Unconditional Coverage (Kupiec)								
Null-Hypothesis	*Correct Exceedances*							
Reject Null	YES	NO	NO	N0	NO	NO	NO	NO
Conditional Coverage (Christoffersen)								
Null-Hypothesis	*Correct Exceedances & Independence of Failures*							
Reject Null	YES	NO	NO	NO	NO	NO	NO	NO

Table 4.3: VaR Backtesting results

By looking at the number of exceedances and failure rates the ARIMA (3.0.2)-EGARCH (1.1) with student-t model has a failure rate closest to 5% at the 5% Value-at-Risk level. The model has a violation ratio of 0.98 which is close to 1 which implies it is a risk conservative model. For the 1% Value-at-Risk level the ARIMA (3.0.2)-GARCH (1.1) with Student-t and the ARIMA (3.0.2)-EGARCH (1.1) with student-t model exhibits a failure rate closest to 1%. However, the ARIMA (3.0.2)-GARCH (1.1) with Student-t distribution has a violation ratio of 0.83 which is more closer to 1 indicating that it produces more accurate value at risk at 1% level of significance.

The ARIMA (3.0.2)-GARCH (1.1) with Student-t and the ARIMA (3.0.2)-EGARCH (1.1) with student-t model overestimates VaR since their failure rate is 0.8 at 1%.All other failure rates are relatively high, suggesting that all the models have projected Value-at-Risk measures that underestimate the risk at hand at 1%. The projected Value at risk measures are all smaller than 5% which means that they overestimate risk and are conservative models.

Based on the results, the model that performed best were the ARIMA (3.0.2)-EGARCH (1.1) with student-t model. The models passed all tests, except for the ARIMA (3.0.2)-GARCH (1, 1) which did not pass test for conditional and unconditional coverage for Value-at-Risk level of 5%.

4.8 CHAPTER SUMMARY

This chapter focused on data presentation, interpretation and discussion of the research findings. From The Bayesian Information Criterion (BIC), the best model for modelling the logarithmic return series of Industrial index of ZSE is the ARIMA (3.0.2). The ARIMA (3.0.2)-EGARCH (1.1) shows presence of asymmetry and leverage effect in industrial index returns of ZSE. Based on the backtesting results, model that performed best were the ARIMA (3.0.2)-EGARCH (1.1) with student-t model with 5% level of significance. The subsequent chapter focuses on conclusion, recommendation and suggestion of further study.

CHAPTER 5

CONCLUSION

5.0 INTRODUCTION

The current chapter serves to give a summary of research, conclusion drawn from the research in conjunction with recommendations on Value at Risk of ZSE using GARCH models. The chapter highlights key findings together with conclusions that address research objectives and finally recommendations on strategic ways to better prepare investors from uncertainties in terms of market risk. The chapter also give suggestions for further study which will add interesting perspective to the area of study.

5.1 CONCLUSIONS

The analysis of the relative performance of GARCH family models in estimating and forecasting VaR of the ZSE:

The research evaluates performance of symmetric and asymmetric GARCH type models. The ARIMA (3.0.2)-EGARCH (1.1) shows presence of asymmetry and leverage effect in industrial index returns of ZSE. Based on the minimum AIC, BIC values and log likelihood values, the best fit model for the series are the ARIMA (3.0.2)-EGARCH (1.1). Thus, asymmetric model ARIMA (3.0.2)-EGARCH (1.1) outperform the symmetric ARIMA (3.0.2)-GARCH (1.1) model on estimating and forecasting VaR of the ZSE.

Evaluating the accuracy of the estimation of VaR calculated by the different GARCH models through backtesting:

The ARIMA(3.0.2)-GARCH(1.1) and ARIMA(3.0.2)-EGARCH(1.1) under normal and t-distribution at 1% and 5% level of significance were used to evaluate accuracy of VaR. Based on the backtesting results, model that performed best were the ARIMA (3.0.2)-EGARCH (1.1) with student-t model with 5% level of significance. From the ARIMA (3.0.2)-GARCH (1, 1) violation ratio, a conclusion can be made that the model is risk conservative. The ARIMA (3.0.2)-GARCH (1, 1) is unacceptable in calculating VaR of ZSE industrial index at 5% significance level based on the conditional and unconditional coverage test.

5.2 RECOMMENDATIONS

The investors and policy makers should incorporate asymmetric Garch models in the estimation and forecasting of Value at Risk as they capture leverage effect and volatility clustering. GARCH-type models are recommended to be used to model and forecast VaR of the ZSE, more specially the ARIMA (3.0.2)-EGARCH (1.1) with student-t model.

5.3 SUGGESTIONS FOR FURTHER STUDY

As recommendations for further study, this research suggest the use of a multivariate GARCH model where it considers other variables such as the inflation and money supply. Furthermore, comparison of ARIMA-GARCH and other models like Multifactor Risk Approach, Risk metrics and Monte Carlo Simulation may add interesting perspectives to the outcomes of this research. Another suggestion is to examine the results further with other backtesting methods for example Pearson's test. The test offers simplicity and it can be applied for more rigorous backtesting. In this research, only two distributions were used the normal and student t-distribution. There are however, numerous distribution that can be used such as: skew t, Normal Inverse Gaussian, Johnson's Su distribution etc.

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APPENDIX I

ADF TEST

```
###############################################################
# Augmented Dickey-Fuller Test Unit Root / Cointegration Test #
###############################################################

The value of the test statistic is: -23.0083 264.6967
```

APPENDIX 2

MODEL FIT OF ARIMA(3,0,2)-GARCH(1.1) WITH NORMAL DISTRIBUTION

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(3,0,2)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000144    0.000422 -0.34174 0.732548
ar1     0.523469    0.355616  1.47201 0.141019
ar2     0.127555    0.307061  0.41541 0.677845
ar3     0.118702    0.049613  2.39255 0.016732
ma1    -0.332757    0.355076 -0.93714 0.348686
ma2     0.002912    0.247861  0.01175 0.990625
omega   0.000010    0.000000 43.99227 0.000000
alpha1  0.501114    0.046230 10.83962 0.000000
beta1   0.497886    0.018875 26.37828 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000144    0.000677 -0.213072 0.831271
ar1     0.523469    0.709921  0.737362 0.460902
ar2     0.127555    0.633191  0.201448 0.840348
ar3     0.118702    0.074153  1.600770 0.109428
ma1    -0.332757    0.711007 -0.468008 0.639779
ma2     0.002912    0.536793  0.005425 0.995671
omega   0.000010    0.000001 14.205737 0.000000
alpha1  0.501114    0.169160  2.962359 0.003053
beta1   0.497886    0.083387  5.970815 0.000000

LogLikelihood : 6077.586

Information Criteria
```

```
------------------------------------

Akaike       -6.9197
Bayes        -6.8916
Shibata      -6.9198
Hannan-Quinn -6.9093

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      6.352 1.173e-02
Lag[2*(p+q)+(p+q)-1][14]   21.997 0.000e+00
Lag[4*(p+q)+(p+q)-1][24]   26.819 1.384e-05
d.o.f=5
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1783  0.6729
Lag[2*(p+q)+(p+q)-1][5]    1.4757  0.7459
Lag[4*(p+q)+(p+q)-1][9]    2.8957  0.7761
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2786 0.500 2.000  0.5976
ARCH Lag[5]    2.0185 1.440 1.667  0.4672
ARCH Lag[7]    2.6751 2.315 1.543  0.5769

Nyblom stability test
------------------------------------
Joint Statistic:  9.9828
Individual Statistics:
mu     0.12987
ar1    0.05445
ar2    0.19535
ar3    0.17770
ma1    0.14135
ma2    0.18396
omega  4.57739
alpha1 0.08693
beta1  0.13681

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        2.1 2.32 2.82
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.2267 0.2201
Negative Sign Bias  0.1027 0.9182
Positive Sign Bias  0.3833 0.7015
Joint Effect        1.7299 0.6303
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     179.7    3.572e-28
2    30     183.4    2.309e-24
3    40     208.0    6.143e-25
4    50     216.0    7.839e-23
```

APPENDIX III

MODEL FIT OF ARIMA(3.0.2)-GARCH(1.1) USING STUDENT-T DISTRIBUTION

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(3,0,2)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000288    0.000223 -1.28980 0.197120
ar1    -0.041544    0.209461 -0.19834 0.842781
ar2     0.637933    0.166029  3.84231 0.000122
ar3    -0.047608    0.051865 -0.91793 0.358655
ma1     0.307157    0.207129  1.48293 0.138094
ma2    -0.468818    0.150607 -3.11286 0.001853
omega   0.000022    0.000005  4.13073 0.000036
alpha1  0.516901    0.108968  4.74360 0.000002
beta1   0.305341    0.098180  3.11001 0.001871
shape   3.094699    0.251274 12.31604 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000288    0.000249 -1.15459 0.248257
ar1    -0.041544    0.187067 -0.22208 0.824251
ar2     0.637933    0.123463  5.16701 0.000000
ar3    -0.047608    0.061485 -0.77430 0.438752
ma1     0.307157    0.184021  1.66914 0.095089
ma2    -0.468818    0.148851 -3.14958 0.001635
omega   0.000022    0.000009  2.48002 0.013138
alpha1  0.516901    0.135458  3.81594 0.000136
beta1   0.305341    0.163855  1.86348 0.062395
shape   3.094699    0.311622  9.93094 0.000000

LogLikelihood : 6371.477
```

```
Information Criteria
------------------------------------

Akaike       -7.2537
Bayes        -7.2225
Shibata      -7.2537
Hannan-Quinn -7.2422

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.2137  0.6439
Lag[2*(p+q)+(p+q)-1][14]    7.3114  0.6147
Lag[4*(p+q)+(p+q)-1][24]   11.4220  0.6361
d.o.f=5
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1853  0.6669
Lag[2*(p+q)+(p+q)-1][5]    1.4783  0.7453
Lag[4*(p+q)+(p+q)-1][9]    3.0906  0.7439
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2038 0.500 2.000  0.6517
ARCH Lag[5]    2.5143 1.440 1.667  0.3684
ARCH Lag[7]    3.0903 2.315 1.543  0.4967

Nyblom stability test
------------------------------------
Joint Statistic:  5.0994
Individual Statistics:
mu     0.10666
ar1    0.24039
ar2    1.00552
ar3    0.04346
ma1    0.52091
ma2    0.70654
omega  1.22649
alpha1 1.88536
beta1  1.44530
shape  2.49703

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        2.29 2.54 3.05
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.6635 0.5071
Negative Sign Bias  0.1577 0.8747
Positive Sign Bias  0.2896 0.7722
```

```
Joint Effect       0.6343 0.8885


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     20.18       0.3835
2    30     34.61       0.2177
3    40     38.02       0.5145
4    50     49.53       0.4518


Elapsed time : 1.217107
```

APPENDIX IV

MODEL FIT OF ARIMA(3.0.2)-EGARCH(1.1) USING STUDENT-T DISTRIBUTION

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(3,0,2)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000286    0.000220  -1.29640 0.194838
ar1     1.407575    0.024007  58.63278 0.000000
ar2    -0.550199    0.017990 -30.58332 0.000000
ar3     0.034934    0.019961   1.75011 0.080099
ma1    -1.140623    0.033329 -34.22296 0.000000
ma2     0.334145    0.026535  12.59275 0.000000
omega  -3.199100    0.760587  -4.20610 0.000026
alpha1  0.017494    0.042932   0.40749 0.683648
beta1   0.673297    0.077847   8.64904 0.000000
gamma1  0.604101    0.079947   7.55624 0.000000
shape   3.133007    0.255645  12.25533 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000286    0.000244  -1.16935 0.242264
ar1     1.407575    0.010094 139.45120 0.000000
ar2    -0.550199    0.013219 -41.62092 0.000000
ar3     0.034934    0.023115   1.51132 0.130706
ma1    -1.140623    0.027036 -42.18849 0.000000
ma2     0.334145    0.029513  11.32211 0.000000
omega  -3.199100    1.258835  -2.54132 0.011044
alpha1  0.017494    0.051441   0.34009 0.733792
beta1   0.673297    0.130047   5.17734 0.000000
```

```
gamma1  0.604101    0.107270   5.63157 0.000000
shape   3.133007    0.307596  10.18547 0.000000

LogLikelihood : 6376.885

Information Criteria
------------------------------------

Akaike       -7.2587
Bayes        -7.2244
Shibata      -7.2588
Hannan-Quinn -7.2460

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.8251 0.36369
Lag[2*(p+q)+(p+q)-1][14]    8.7993 0.01849
Lag[4*(p+q)+(p+q)-1][24]   13.0198 0.39586
d.o.f=5
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.01607  0.8991
Lag[2*(p+q)+(p+q)-1][5]   1.13298  0.8289
Lag[4*(p+q)+(p+q)-1][9]   2.47539  0.8413
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2079 0.500 2.000  0.6484
ARCH Lag[5]    2.1051 1.440 1.667  0.4485
ARCH Lag[7]    2.5819 2.315 1.543  0.5957

Nyblom stability test
------------------------------------
Joint Statistic:  5.4301
Individual Statistics:
mu     0.1096
ar1    0.6279
ar2    0.8753
ar3    0.6613
ma1    0.4481
ma2    1.0076
omega  1.5400
alpha1 0.3913
beta1  1.4601
gamma1 0.1710
shape  2.5207

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        2.49 2.75 3.27
Individual Statistic:   0.35 0.47 0.75
```

```
Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.54196 0.5879
Negative Sign Bias 0.35754 0.7207
Positive Sign Bias 0.09179 0.9269
Joint Effect       0.73348 0.8653


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     19.25      0.44093
2    30     38.78      0.10591
3    40     42.62      0.31797
4    50     71.43      0.01992


Elapsed time : 3.243
```

APPENDIX V

MODEL FIT OF ARIMA (3.0.2)-IGARCH(1.1) USING STUDENT-T DISTRIBUTION

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : iGARCH(1,1)
Mean Model      : ARFIMA(3,0,2)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000298    0.000219 -1.36100 0.173513
ar1     1.392845    0.321821  4.32801 0.000015
ar2    -0.523443    0.302838 -1.72846 0.083906
ar3     0.029355    0.051341  0.57176 0.567482
ma1    -1.129962    0.320783 -3.52252 0.000427
ma2     0.317308    0.210393  1.50817 0.131512
omega   0.000024    0.000006  3.88575 0.000102
alpha1  0.679347    0.091042  7.46189 0.000000
beta1   0.320653          NA       NA       NA
shape   2.811518    0.152730 18.40843 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000298    0.000220 -1.35784 0.174514
ar1     1.392845    0.203731  6.83670 0.000000
ar2    -0.523443    0.218920 -2.39103 0.016801
ar3     0.029355    0.050794  0.57791 0.563322
```

```
ma1    -1.129962    0.199639 -5.66002 0.000000
ma2     0.317308    0.150906  2.10269 0.035493
omega   0.000024    0.000009  2.59742 0.009393
alpha1  0.679347    0.149244  4.55193 0.000005
beta1   0.320653          NA       NA       NA
shape   2.811518    0.178229 15.77476 0.000000

LogLikelihood : 6370.537

Information Criteria
------------------------------------

Akaike       -7.2537
Bayes        -7.2257
Shibata      -7.2538
Hannan-Quinn -7.2434

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.3381  0.5609
Lag[2*(p+q)+(p+q)-1][14]    7.4930  0.4941
Lag[4*(p+q)+(p+q)-1][24]   11.6426  0.6027
d.o.f=5
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1349  0.7134
Lag[2*(p+q)+(p+q)-1][5]    1.4972  0.7406
Lag[4*(p+q)+(p+q)-1][9]    3.0974  0.7428
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2422 0.500 2.000  0.6226
ARCH Lag[5]    2.5235 1.440 1.667  0.3667
ARCH Lag[7]    3.1168 2.315 1.543  0.4918

Nyblom stability test
------------------------------------
Joint Statistic:  4.5654
Individual Statistics:
mu     0.1033
ar1    0.6548
ar2    0.7998
ar3    0.5746
ma1    0.5318
ma2    0.9668
omega  1.1048
alpha1 0.8282
shape  2.2967

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        2.1 2.32 2.82
```

```
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.64186 0.5210    
Negative Sign Bias 0.08488 0.9324    
Positive Sign Bias 0.10795 0.9140    
Joint Effect       0.60325 0.8957    


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     17.06     0.585773
2    30     52.36     0.004965
3    40     47.28     0.170451
4    50     51.87     0.362513


Elapsed time : 0.777468
```

APPENDIX VI

MODEL FIT OF ARIMA (3.0.2)-IGARCH(1.1) USING NORMAL DISTRIBUTION

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : iGARCH(1,1)
Mean Model      : ARFIMA(3,0,2)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000146    0.000420 -0.348393 0.727545
ar1     0.525740    0.340769  1.542806 0.122878
ar2     0.124873    0.293225  0.425862 0.670208
ar3     0.118796    0.049485  2.400627 0.016367
ma1    -0.335192    0.339093 -0.988496 0.322910
ma2     0.005225    0.235996  0.022141 0.982336
omega   0.000010    0.000000 33.409226 0.000000
alpha1  0.501818    0.013639 36.794176 0.000000
beta1   0.498182          NA        NA       NA

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000146    0.000599 -0.244466  0.80687
ar1     0.525740    0.947535  0.554851  0.57900
ar2     0.124873    0.822441  0.151833  0.87932
```

```
ar3     0.118796    0.076282  1.557336  0.11939
ma1    -0.335192    0.954263 -0.351257  0.72540
ma2     0.005225    0.690967  0.007562  0.99397
omega   0.000010    0.000001 11.449369  0.00000
alpha1  0.501818    0.029998 16.728540  0.00000
beta1   0.498182          NA        NA       NA

LogLikelihood : 6077.613

Information Criteria
------------------------------------

Akaike       -6.9209
Bayes        -6.8959
Shibata      -6.9209
Hannan-Quinn -6.9117

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic   p-value
Lag[1]                       6.363 0.0116517
Lag[2*(p+q)+(p+q)-1][14]    21.996 0.0000000
Lag[4*(p+q)+(p+q)-1][24]    26.814 0.0000139
d.o.f=5
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1768  0.6741
Lag[2*(p+q)+(p+q)-1][5]    1.4766  0.7457
Lag[4*(p+q)+(p+q)-1][9]    2.8983  0.7756
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2795 0.500 2.000  0.5970
ARCH Lag[5]    2.0196 1.440 1.667  0.4669
ARCH Lag[7]    2.6776 2.315 1.543  0.5764

Nyblom stability test
------------------------------------
Joint Statistic:  8.5092
Individual Statistics:
mu     0.13009
ar1    0.05437
ar2    0.19522
ar3    0.17788
ma1    0.14149
ma2    0.18404
omega  4.55900
alpha1 0.14048

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.89 2.11 2.59
Individual Statistic:   0.35 0.47 0.75
```

```
Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.2255 0.2205    
Negative Sign Bias  0.1043 0.9170    
Positive Sign Bias  0.3798 0.7041    
Joint Effect        1.7269 0.6310    


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     179.0    4.771e-28
2    30     183.4    2.378e-24
3    40     207.9    6.379e-25
4    50     218.1    3.346e-23


Elapsed time : 0.521744
```

APPENDIX VII

ARIMA (3.0.2)-GARCH(1.1) UNDER NORMAL DISTRIBUTION BACKTEST AT 95%

```
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1200
Data:

==========================================
alpha:                          1%
Expected Exceed:        12
Actual VaR Exceed:      16
Actual %:                       1.3%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        1.219
LR.uc Critical:         6.635
LR.uc p-value:          0.269
Reject Null:            NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        1.652
LR.cc Critical:         9.21
LR.cc p-value:          0.438
Reject Null:            NO
```

APPENDIX VIII

ARIMA (3.0.2)GARCH (1.1) UNDER NORMAL DISTRIBUTION BACKTEST AT 5%

```
VaR Backtest Report
===========================================
Model:                        sGARCH-norm
Backtest Length:       1200
Data:

==========================================
alpha:                        5%
Expected Exceed:       60
Actual VaR Exceed:     44
Actual %:                     3.7%

Unconditional Coverage (Kupiec)
Null-Hypothesis:       Correct Exceedances
LR.uc Statistic:       4.93
LR.uc Critical:        3.841
LR.uc p-value:         0.026
Reject Null:           YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:       Correct Exceedances and
                              Independence of Failures
LR.cc Statistic:       10.028
LR.cc Critical:        5.991
LR.cc p-value:         0.007
Reject Null:           YES
```

APPENDIX IX

ARIMA (3.0.2)GARCH(1.1) UNDER STUDENT-T BACKTEST AT 5%

```
VaR Backtest Report
===========================================
Model:                        sGARCH-std
Backtest Length:       1200
Data:

==========================================
alpha:                        5%
Expected Exceed:       60
Actual VaR Exceed:     59
Actual %:                     4.9%

Unconditional Coverage (Kupiec)
Null-Hypothesis:       Correct Exceedances
LR.uc Statistic:       0.018
LR.uc Critical:        3.841
LR.uc p-value:         0.894
Reject Null:           NO
```

```
Conditional Coverage (Christoffersen)
Null-Hypothesis:       Correct Exceedances and
                                      Independence of Failures
LR.cc Statistic:       0.432
LR.cc Critical:        5.991
LR.cc p-value:         0.806
Reject Null:           NO
```

APPENDIX X

ARIMA (3.0.2)GARCH (1.1) UNDER STUDENT-T   BACKTEST AT 1%

```
 VaR Backtest Report
===========================================
Model:                        sGARCH-std
Backtest Length:       1200
Data:

==========================================
alpha:                        1%
Expected Exceed:       12
Actual VaR Exceed:     10
Actual %:                     0.8%

Unconditional Coverage (Kupiec)
Null-Hypothesis:       Correct Exceedances
LR.uc Statistic:       0.357
LR.uc Critical:        6.635
LR.uc p-value:         0.55
Reject Null:           NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:       Correct Exceedances and
                                      Independence of Failures
LR.cc Statistic:       3.668
LR.cc Critical:        9.21
LR.cc p-value:         0.16
Reject Null:           NO
```

APPENDIX XI

EGARCH BACKTEST UNDER NORMAL DISTRIBUTION AT 1%

```
VaR Backtest Report
===========================================
Model:                        eGARCH-norm
Backtest Length:       1200
Data:

==========================================
alpha:                        1%
Expected Exceed:       12
Actual VaR Exceed:     16
Actual %:                     1.3%
```

```
Unconditional Coverage (Kupiec)
Null-Hypothesis:       Correct Exceedances
LR.uc Statistic:       1.219
LR.uc Critical:        6.635
LR.uc p-value:         0.269
Reject Null:           NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:       Correct Exceedances and
                                      Independence of Failures
LR.cc Statistic:       1.652
LR.cc Critical:        9.21
LR.cc p-value:         0.438
Reject Null:           NO
```

APPENDIX XII

EGARCH BACKTEST UNDER NORMAL DISTRIBUTION AT 1%

```
VaR Backtest Report
===========================================
Model:                        sGARCH-norm
Backtest Length:       1200
Data:

==========================================
alpha:                        5%
Expected Exceed:       60
Actual VaR Exceed:     46
Actual %:                     3.8%

Unconditional Coverage (Kupiec)
Null-Hypothesis:       Correct Exceedances
LR.uc Statistic:       3.727
LR.uc Critical:        3.841
LR.uc p-value:         0.054
Reject Null:           NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:       Correct Exceedances and
                                      Independence of Failures
LR.cc Statistic:       4.511
LR.cc Critical:        5.991
LR.cc p-value:         0.105
Reject Null:           NO
```

APPENDIX XIII

EGARCH BACKTEST UNDER STUDENT-T DISTRIBUTION AT 5%

```
VaR Backtest Report
===========================================
Model:                        eGARCH-std
```

```
Backtest Length:       1200
Data:

==========================================
alpha:                        5%
Expected Exceed:       60
Actual VaR Exceed:     47
Actual %:                     3.9%

Unconditional Coverage (Kupiec)
Null-Hypothesis:       Correct Exceedances
LR.uc Statistic:       3.193
LR.uc Critical:        3.841
LR.uc p-value:         0.074
Reject Null:           NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:       Correct Exceedances and
                                    Independence of Failures
LR.cc Statistic:       3.688
LR.cc Critical:        5.991
LR.cc p-value:         0.158
Reject Null:           NO
```

APPENDIX XIV

EGARCH BACKTEST UNDER STUDENT-T DISTRIBUTION AT 1%

```
VaR Backtest Report
==========================================
Model:                        eGARCH-std
Backtest Length:       1200
Data:

==========================================
alpha:                        1%
Expected Exceed:       12
Actual VaR Exceed:     9
Actual %:                     0.8%

Unconditional Coverage (Kupiec)
Null-Hypothesis:       Correct Exceedances
LR.uc Statistic:       0.829
LR.uc Critical:        6.635
LR.uc p-value:         0.362
Reject Null:           NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:       Correct Exceedances and
                                    Independence of Failures
LR.cc Statistic:       0.965
LR.cc Critical:        9.21
LR.cc p-value:         0.617
Reject Null:           NO
```