A1.
   a. According to their sources, how are errors classified? Explain briefly.
   b. Determine how many correct digits $\bar{x} = 1299.79$ has, if $x = 1300.00$ Justify.
   c. If $\bar{x} = 0.0068457$ has 3 correct digits, determine the possible values of $x$. Justify.

A2. Apply Newton's method, to find the root in the first quadrant of the system

$$x_1^2 + x_2^2 = 1$$
$$x_1^2 = x_2^2 - \frac{1}{2}$$

Carry out one iteration. Take $(1, 3)'$ as initial approximation.

According to what you can observe, in practice, is the method converging? Justify.

A3. In a financial market where 3 stocks are traded and there are 3 possible economic situations in the near future, the portfolio that the investor Mr. John Smith should buy today to meet his future consumption needs, can be found by solving the system $Ax = b$, where

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix}.$$

a. Carry out two steps of Gauss-Seidel method.

b. In practice, is the method converging? Explain.

c. Theoretically, does the method converge in this case? Justify.

A4. Solve the system $Ax = b$, where

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}.$$

If you performed well, the found vector \( x \) is solution of the system. What does this mean?

**A5.** In the following table, \( x \) denotes the prices of some item in a specific shop in 2011, 2013 and 2015, and \( f(x) \) represents the corresponding number of articles that were sold (in thousands). Assume that these values are exact:

<table>
<thead>
<tr>
<th>( x )</th>
<th>10</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>22</td>
<td>19</td>
<td>14</td>
</tr>
</tbody>
</table>

a. Obtain the interpolating polynomial \( p_2(x) \) at the given points.  \[12\]
b. Verify that, in fact, \( p_2(x) \) interpolates \( f(x) \) at these points. \[4\]
c. Find approximately the number of articles that were sold in 2014, when the price was 13. In other words, estimate \( f(13) \). \[3\]
d. Bound the error in the preceding estimation if \( f \) is 3 times differentiable and \( |f^3(x)| \leq M, x \in [10, 15] \). \[6\]
END OF QUESTION PAPER