BINDURA UNIVERSITY OF SCIENCE EDUCATION
PHYSICS DEPARTMENT

PH 205 – VECTOR METHODS AND ELECTROMAGNETISM

INSTRUCTIONS APR 201 **TIME: 3 HOURS**

Answer ALL parts of question one in Section A and any THREE questions from Section B. Section A carries 40 marks and each question in Section B carries 20 marks.

Physical Constants

Universal Gravitation Constant, $G = 6.7 \times 10^{-11}$ Nm^{2}kg^{-2}
Acceleration due to gravity, $g = 9.8$ ms^{-2}
Permittivity of free space $\varepsilon_\circ = 8.85 \times 10^{-2}\, Fm^{-1}$
Permeability of free space $\mu_\circ = 4\pi \times 10^{-7}\, Hm^{-1}$
Velocity of light, c = 3.00 x 10^8 ms^{-1}

SECTION A

1(a) (b) Find the electric intensity at every point of space of a sphere of radius R with surface density of charges equal to $\frac{Q}{4\pi R^2}$ where Q is the total charge. [4]

(b) Given that $\bar{A} = 2\bar{i} + 4\bar{j} + 2\bar{k}$ and $\bar{B} = 3\bar{i} + 4\bar{j} - 11\bar{k}$ verify whether vectors $\bar{A}$ and $\bar{B}$ are perpendicular [4]

(c)(i) Show, starting from the appropriate Maxwell equation, that the normal component of $\bar{B}$ are continous across a plane boundary between media with different magnetic properties [5]

(ii) Explain what this implies for the conservation of the lines of $\bar{B}$ [3]

(d) Show that $\nabla \bullet \left(\frac{\bar{r}}{r^3}\right) = 0$ [3]

(e)(i) Briefly explain the physical meaning of the poynting vector $\bar{S} = \bar{E} \times \frac{\bar{B}}{\mu_o}$ for time varying fields. [4]

(ii) State units of the poynting vector [2]

(f) Find the wavelength λ, skin depth δ, and the refractive index n, of an electromagnetic wave of angular frequency $\omega = 2\pi \times 10^{10} s^{-1}$ propagating through aluminium of resistivity $\sigma = 3.53 \times 10^{7} \Omega m^{-1}$ [9]

(g) Use the Cartesian form of ∇ to show that, when $\bar{A}$ is a vector field

$$\nabla \bullet (\nabla \times \bar{A}) = 0 \ .$$ [7]

SECTION B

2(a) Evaluate $\iint_s \nabla \times \bar{A} \bullet \bar{n} ds$ where $\bar{A} = (x^2 + y - 4)\bar{i} + 3xy\bar{j} + (2xz + z^2)\bar{k}$ and 's' is a surface of a hemisphere $x^2 + y^2 + z^2 = 16$ above the xy plane. [5]

(b)(i) Simplify the expression $\delta\left(\sqrt{(x^2+1)} - x - 1\right)$. [5]

(ii) Simplify the function $\delta(\sin x)$ and sketch the graph. [10]

3(a) State four Maxwell's equations in their integral form [4]

(b) As applied to electromagnetic waves state three modified Maxwell's equations and explain the modifications [6]

Hence prove that one of the electromagnetic wave equations is

$$\nabla^2 \bar{B} = \varepsilon_{\circ}\mu_{\circ} \frac{\partial^2 \bar{B}}{\partial t^2}$$ [10]

4(a) The Gauss law for electricity can be written in the form

$$\oint_s \bar{E}(\bar{r}) \bullet d\bar{S}(\bar{r}) = \frac{q'}{\varepsilon_{\circ}}$$

(i) Explain the symbols used in this expression and; [3]

(ii) explain what the expression means [2]

4(b) Use Gauss law to determine the electric field and the potential at a distance r from the centre of a uniformly charged sphere of radius R and total charge Q. [15]

5(a)(i) Write down Maxwell's four equations in their differential form and; [3]

(ii) Name the observational law which each equation represents [2]

(b) Using Maxwell equations, show that
(i) a time-dependent magnetic field cannot exist without an electric field [3]

(ii) a uniform electric field cannot exist in the presence of a time-dependent magnetic field [3]
(iii) inside an empty cavity a uniform electric (or magnetic) field can be time-dependent. [3]

(c) (i) Explaining all necessary assumptions, use appropriate Maxwell equations to derive the wave equation

$$\nabla^2 \bar{E} - \sigma\mu \frac{\partial \bar{E}}{\partial t} - \varepsilon\mu \frac{\partial^2 \bar{E}}{\partial t^2} = 0 \qquad [5]$$

(ii) Give the simplified form of this equation for waves in vacuum. [1]

6(a) Demonstrate from $\bar{F} = q\bar{v} \times \bar{B}$ that $\bar{B} = \frac{\bar{F} \times \bar{v}}{qv^2} + k(\bar{v} \bullet \bar{B})\bar{v}$ where $k = \frac{1}{v^2}$. [6]

(b) State the Faraday's law of electromagnetic induction. Hence show the following Maxwell's equation

$$\text{curl}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad [8]$$

(c) Derive the continuity equation. [6]

END OF EXAMINATION PAPER