AN ANALYSIS OF STUDENTS’ CONCEPTUAL UNDERSTANDING OF ADVANCED LEVEL GROUP CONCEPTS AT MUTONDWE HIGH SCHOOL IN Mt DARWIN DISTRICT

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DECLARATION FORM

I declare that: **AN ANALYSIS OF STUDENTS’ CONCEPTUAL UNDERSTANDING OF ADVANCED LEVEL GROUP CONCEPTS AT MUTONDWE HIGH SCHOOL IN Mt DARWIN DISTRICT** is my own work, has not been submitted before for any degree or examination in any other university and that all the sources used or quoted have been indicated and acknowledged as complete references.

Name: MUSVEHE FARAI

Signed: ……………………………………………………………………..
DEDICATION

I dedicate this research project to my wife Nomatter and children. You are very important to me.
ACKNOWLEDGEMENT

The success of this project is a result of contributions and assistance I got from various people to whom I am much indebted. To start with, I would like to express my sincere gratitude to my wife, for the moral and financial support she always rendered to me whenever I was in need. Secondly I would want to extend my indebtedness to my supervisor, Dr Dziva, who helped so much in development of this research project. Her guidance and advisory contribution were highly indispensable, without her input, this work would not have been what it is like right now. I would also want to thank the deputy head teacher of Mutondwe High School in Mt Darwin District for granting me permission to conduct my research project at her school. This leave me with no option but to spontaneously convey my gratitude to Mutondwe High School staff and students for their support and participation in this research project. It was indeed a sacrifice on their part given the pressure under which they were operating due to the impending Advanced level final examinations. Lastly I would like to appreciate all my college mates and all those behind the science for their contributions I cannot singly mention not because they were not important but they were so pertinent that few words would be an understatement. Over and above, I would like to thank my God, the creator, for allowing me to achieve all that I have managed to achieve.
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ABSTRACT

This study analysed the students’ conceptual understanding of the concept of Groups at Advanced Level using the Action-Process-Object-Schema (APOS) theory. This was done to show the levels of understanding displayed by different students thus, the research sought to address the way how students construct knowledge as they will be learning groups concepts. The main purpose was to establish the levels of operation individual students demonstrate when learning Groups and find possible intervention measures for these learners to improve their conceptual understanding of Groups to improve their overall performance in mathematics.

The researcher used Mutondwe high School in Mt Darwin as a case study where 10 Advanced level students were randomly selected. The selected students were taught the concept of Groups for four days (four lessons) using Activities-Classroom discussions-Exercises (ACE) Strategy before a test which comprised simple definitions of Group concepts, properties of groups was administered. The test saw the concept of Groups broken down into APOS levels (preliminary genetic decomposition). The test was then marked and the outcomes where categorized according to the preliminary genetic decomposition. Two days later, the researcher conducted follow-up interviews guided by the learners test responses. The interviews sought to gain a deeper analysis of students’ conceptual understanding of Groups and hence work out possible intervention measures together with the students.

It was found out that most students operated at the process level and very few at object level whereby they could not construct tables to represent a group perfectly and convincingly. Students operating at object level could not determine commutative groups using their properties without using rote –learning or guess work, thus they could not exude deep conceptual understanding of Group concepts. However, all students could define Group by stating the properties and also give the order of a group which indicated that they were all operating at least the action level.

The subsequent interviews suggested that since pupils operate at different levels of APOS, it is necessary to try and incorporate teaching methodologies that allow the teacher to pay particular attention to individual differences as well as problem solving approach. Giving students more practice work has been put forward as a way to help students cope with some of the complexities of Group concepts. The researcher therefore recommends that the ACE
approach can be used to assist learners improve on their conceptual understanding of Group concepts at Advanced level.
CHAPTER ONE

Putting the problem into context

1.0 Introduction

In the chapter the following issues are going to be addressed: the background of study of study to put the research problem into context. A statement of the problem followed by, research questions, significance of study, assumptions, delimitation, limitations and definition of terms are given. Lastly summary of the chapter is outlined.

1.1 Background to the study

Mathematics has been regarded as a crucial subject of learning that is helping economies and transformation of societies technologically. Then the teaching and learning of concepts in Mathematics such as Groups at Advanced level is of paramount importance. Groups are a learning area that involves axioms such as associativity, closure, identity element and inverse. It also involves the commutative (abelian group), commutative groups among other things. It appears that no breakthrough can be attained in the development of Mathematics without critically analysing how students acquire mathematical concepts.

The application of mathematics concepts continues to be of relevance even in various subjects studied at Advanced Level (highest level of secondary school). In view of this, mathematics is given more time to the extent that it is mandatory that this subject is taught every day in the school week and having at least one double lesson per week or cycle. Not only is mathematics a basic subject in the new curriculum, but it is also a pre-requisite for one to enrol in most of the courses that are offered at various Higher and Tertiary Colleges like universities, colleges and apprenticeship training institutions.
It is against this background that an analysis of students` conceptual understanding of various often problematic concepts becomes a key undertaking towards the improvement of students `performance in mathematics as subject. Concepts that are normally poorly understood by students include geometry, transformations and groups. In line with this, the researcher seeks to make an analysis of learners` conceptual understanding of group concepts at Advanced level with reference to the Action-Process-Object-Schema (APOS) framework which is a consequence of constructivist theory of learning.

1.2 Statement of the problem

The APOS theory is a framework used by many researchers to describe how learners acquire new concepts. This research seeks to analyze learners` conceptual understanding of group concepts using APOS theory and try to recommend teaching and learning approaches which promote both conceptual and procedural understanding of Group concepts. The analysis will be accomplished by classifying students according to the levels of the APOS theory that is (action, process, object and schema). Therefore it is the goal of this study to embrace the APOS theory in order to have more insight on ways in which learners understand group concepts with respect to their different levels of operation. Dubinsky (1991) describes a teacher as someone with deep understanding of the concepts that is in this context, he or she is someone who has both procedural and conceptual understanding of Group concepts. This will in turn be transferred to the learner. The teacher therefore should have the expertise to employ the APOS theory as a tool to objectively explain the students` difficulties or challenges in conceptual understanding of Group concepts.
1.3 RESEARCH QUESTIONS

The study sought to answer the following questions:

- What levels according to APOS do Advanced level students’ understanding of group concepts’ can be classified?
- How do these levels reflected in their understanding of Group concepts?
- What strategies could be adopted to improve learners’ conceptual understanding of Group concepts at various levels of operation identified using APOS theory?

1.4 Significance of the study

The knowledge generated from the study may be useful to the following:

**The teacher**

The researcher findings will assist the Advanced level Maths teachers in assessing the teaching and learning of Maths in general and Groups in particular. The teacher will also be in a position to familiarise with recently adopted strategies of teaching Groups at Advanced level.

**The learner**

The study will provide learners with appropriate skills needed in carrying out research. It will also assist the learners with information concerning teaching and learning of Groups at Advanced level.
The Publishing house

It is believed that results of the research will help textbook publishers in effectively supporting the teaching of the new Advanced level topic, Groups. They would produce textbooks that are relevant to the teaching and learning of Groups at Advanced level.

Ministry of Primary and Secondary Education

The study will help the curriculum developers to be aware of the strategies being adopted in the teaching and learning of Groups. This will help the Ministry to aid schools by providing necessary teaching resources so as to improve teaching and learning processes.

1.5 Delimitations

The research will be carried out using Advanced level class at Mutondwe High School in Mt Darwin District in Mashonaland Central Province. The class comprises of ten learners, eight boys and two girls. It is hoped that the sample of these learners doing Advanced level mathematics will be generalised and represent the whole District, Province and Zimbabwe at large.

1.6 Limitations

The research is limited to the student’s understanding of group concepts due to absenteeism caused by them not paid up their school fees.

Preparations of lesson material were always disturbed, due to persistent power cuts.

Limited number of computers affected the smooth delivery of the lessons.
1.7 Assumptions

It is assumed that advanced level mathematics students have knowledge of sets which is considered basic for the learning of group concepts. It is further assumed that the students have knowledge of basic operations in mathematics and they are capable of developing conceptual understanding of mathematical concepts presented in class.

1.8 Definition of terms

Groups: a group is \((G,*)\) is a set \(G\) together with a law of composition \((a, b) \rightarrow\)that satisfies the following axioms, associative, identity, inverse and closure.

Learning: is the process of acquiring new, or modifying existing knowledge, behaviours, skills, values or preferences(Bruff, 2009).

Teaching: those moments or sessions where we make specific interventions to help people learn particular things(Jeffs and Smith, 2018).

Strategy: concerned with the actions and resources needed to achieve specific objectives(Hart, 2009).

1.9 Summary

The chapter dealt on the background of study which pointed out the crucial role mathematics plays in the general life of students. It also highlighted the importance of conceptual understanding when students learn group concepts and mathematics in general. The chapter
also outlined the statement of the problem under study. Research questions, significance of study, delimitations, limitations and assumptions, of the study are given. Lastly, definitions of terms, and the chapter summary is provided. Chapter two will reviews selected literature on an analysis of students’ conceptual understanding of Advanced level Group concepts, chapter three will describe the methodology to be used in the research, chapter four will be on data presentation and analysis and lastly chapter five will provide the summary, conclusions and recommendation of the study..
CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.0 INTRODUCTION

In the chapter the researcher is going to do an exploration of related literature on the analysis of conceptual understanding of groups at advanced level. The theory used to analyse the conceptual understanding of groups by students will be outlined. The researcher will go further to discuss groups as a concept and how students learn and understand the concept. Finally, he will look at the relationship between the introductory genetic decomposition of the concept of groups as well as the students’ constructs. The researcher will highlight some relevant studies on how teaching and learning of groups is influenced by the students’ understanding.

2.1 THEORETICAL FRAMEWORK

The study will be fundamentally concerning the theory of ACTION-PROCESS-OBJECT-SCHEMA (APOS) which was propounded by Dubinsky (1984) and having its roots in constructivist theory by Piaget (1964). This theory informs and guides data collection and analysis (Brijlall and Maharaji, 2010). It describes how mathematics concepts can be learnt (Dubinsky & Wilson, 2013). This theory was initially introduced in an effort to try and give the processes that happen when one tries to learn a novel mathematical concept. In this framework, leaners mentally formulate their understanding of a given mathematical idea, APOS theory makes use of ACE teaching cycle which is a teaching strategy comprising three repeated activities or processes.
These constituencies are: Activities, Classroom discussion, Exercises. The initial step in ACE pedagogical strategy involves activities that are meant to conjure up students’ development of the mental structures but they are not performed in the classroom (Asiala et al, 1996). The next step takes place in the classroom where learners reflect on the activities that they were doing under the supervision of the teacher. This is accomplished by means of different types of interactions which happen as they discuss. The final step occurs when students are given written work in the form of class exercise or homework, the purpose being to consolidate the knowledge gained from the activities and students’ experiences in the classroom. The use of this pedagogical approach has proved to be very effective in assisting learners to understand the mathematical concept learnt and ultimately enhance their capability to solve required mathematical problems distinct from the ones learnt (Dubinsky, 1991).

It is crucial to note that APOS theory was basically developed on the assumptions that:

- Mathematical knowledge is based on the ability to react to a mathematical problem situation and how solutions are obtained through constructing mental structures in relation to their social context (Dubinsky, 1991).
- Concepts in mathematics cannot be acquired in a direct way since the process of learning depends on the processed mental structures (Dubinsky and Wilson, 2013). In order to come up with meaning of any mathematical concept, one applies mental structures (Piaget, 1964).

The implication of these assumptions is that appropriate mental structures are a requisite feature for one to be able to learn a given mathematical concept. It is also necessary to consider learners background when choosing the type of problems to give him or her in order maintain harmony between what the learner already conceives and the new concept to be learnt. This avoids confusion and enhances the learners’ mastery of the mathematical
concept. However, the unavailability of necessary mental structures in the student can make it difficult for the learner to grasp the new concept. It is necessary for the teacher to use pedagogical approaches that can help students to formulate the suitable mental structures and guiding them to use these structures to construct their own understanding of mathematical concept (Piaget, 1964).

The APOS theory makes it clear that conceptual understanding takes place in four stages which are: object, process, action and schema.

**Action**-refers to a change of objects which are viewed by individuals as fundamentally external and as requiring instruction which occurs in stages showing how an operation is performed. It needs specific instructions and to perform each stage of the transformation clearly (Dubinsky and Wilson, 2013). For instance, at an action level of understanding the group \((z, +)\), the learner may need the concept of sets and numbers in order to draw a simple table in order to give the correct sets of elements in the group \((z,+)\) which is process stage.

**Process** stage-this refers to a mental construction that is made by an individual when an action is repeated and reflected upon it. When an individual repeats an action, this may be interiorized into mental process (Dubinsky, 2010). Brijlall and Maharaji (2010) argues that a process as a mental structure doing the same operation as the action, but fully in the mind of the individual. The learner can be capable of doing the transformation without having to do the steps clearly. A student with a process understanding can consider performing the same action without using external stimuli. Dubinsky (2010) goes on to say that an individual might decide to perform a process without doing it, he can decide to reverse it and compare the process with another process. By so doing the learner will be able to give the required solution set.
Object refers to a structure from a process where the individual becomes cognisant of the process as a whole and note that change can act on it. If a student can appreciate this and builds the changes, so it can be said that the learner has encapsulated the process into a cognitive object (Dubnisky, 2010). Considering the above given example, the individual is able to compare, relate the solution set to concept of inequalities, and can create linkages between concept, thus, has encapsulated the process into objects.

According to Dubnisky, (1991), Schema refers to an organized and linked logical framework of an individual’s collection of actions, processes, objects and other related schemas. The linkage is due to the fact that it gives an individual with a method of deciding, when presented with a mathematical problem, whether the schema applies or not (Dubnisky, 2010). It is maintained that this framework happens in an individual’s mind which is presented with a problem situation that involves the concept. At this stage, one can apply the concept in real life situations. Brijlall and Maharaji (2010) notes that clarifications presented by an APOS analysis are limited to explanations of the thinking which an individual might be capable of. Brijlall and Maharaji (2010) further observes that, it does not necessarily mean that if an individual possesses a certain mental structure, then someone can apply it in a given condition as this can depend on different factors.

The main objective of APOS analysis is to provide possible teaching approaches for helping students learn it. Theoretical analysis proposes that a genetic disintegration is a set of mental formations that a student might create in order to comprehend the mathematical concept being learnt (Dubinsky, 2010). Basing on the theoretical framework, the research intends to address issues such as students’ conceptual understanding of linear programming concept, relationship between students’ mental creations and genetic decomposition, how the understanding of a concept influences teaching methodologies and lastly, weaknesses of linear programming concept.
2.2 GROUP CONCEPT

According to Gordon (2015) group theory is a theory that concerns the characteristics and properties of a group. Gallian (2015) also observed that a group is an algebraic structure formed by a set and a binary operation that satisfies some properties. The definition of a group given by Gallian (2015), refers to a non-empty set $G$ with a binary operation $*$ which is said to be a group if the following axioms are satisfied:

i) $a*b \in G$, for all $a \in G$ (closure).

ii) There is $e \in G$ such that $e*a = a*e = a$, for all $a \in G$.

iii) For every $a \in G$, there is $1/a \in G$ such that $a*1/a = 1/a*a = e$.

iv) $(a*b)*c = a*(b*c)$, for all $a, b, c \in G$.

Many students are finding it difficult to learn group concepts due to their abstractness. A certain teaching strategy in teaching and learning of Group theory was needed so as to lessen the burden. Some specialists applied different methods to teach group theory. Cornock (2015) recommended teaching Group theory using Rubik’s cubes and students gave positive responses to the teaching methods and the use of cubes. Gordon (2015) also offered the teaching of Group theory using geometry approach and showed commercial software packages that can be used to augment student’s understanding.

Hazan cited by Gordon (2015) pronounced strategies in which learners deal with Group theory concepts by making these concepts mentally manageable. This implies that, the ways in which learners apprehend Group theory concepts are scrutinized through the exercise of lessening the abstraction level. Concepts from Group theory are very abstract, therefore, to diminish the level of abstraction, technology has been useful. According to Rosenbaum (2010), software has become an accepted tool in Group theory. Cornock (2015) cited that many software programmes have been developed to simplify students’ learning,
explore the properties of the group. These include Group Explorer, GAP, Magma, Cayley, Kali and Tess. According to Gordon (2015), GAP (groups, algorithms and programming) is a computer algebra system for computational discrete algebra with particular emphasis on computational group theory. Magma is a large, well-supported software platform aimed for computations in algebra, number theory, algebraic geometry and algebraic combinations, (Juniatic and Budayasa,2017). This software offers a mathematically demanding environment for defining and working with structures such as groups.

Kali and Tess is a program to form a pattern of conception of a group, such as a cyclic group, dihedral group, frieze group or wallpaper group,(Cornock,2015). In the program, students are asked to plan appropriate patterns of each type frieze group .The exercise will aid learners to become more familiar with local culture with its features and get to understand the variance of symmetrical patterns. This activity of learning intensifies students’ motivation in learning group theory as they sense the application of the theories learned in life.

2.3 LINKAGE BETWEEN STUDENTS’ MENTAL CONSTRUCTIONS AND GENETIC DECOMPOSITION

Arnon et al (2013) indicate that a genetic disintegration is a tool by which scholars attempt to have an understanding of how students learn particular mathematical concepts and to enlighten the reasons behind student complications which can exist in the form of a hypothetical model of mental formations that a student require to make in order to learn a particular mathematical concept. Mathematical concepts include defining a group, identifying elements of a group, for example the identity and inverse elements and also formulating tables to illustrate elements of a group defined by a specified binary operation problems which are constructed in the student’s mind .According to Gallian(2010), students make sense of mathematical concepts by creating and using certain mental constructions.
According to APOS theory these are called the stages of learning of mathematical models (Piaget & Garci, 1983). The genetic decomposition consists of a description of the actions that a student needs to perform on existing mental objects and continues to include explanations of these actions are interiorized into processes (Brijlall and Maharaj, 2010). While at this point, the concept is perceived as something one does. The process is encapsulated into mental objects so as to be conceived as an entity and something that can be transformed. It is exclusively possible that a concept may consist of numerous different actions, processes and objects. Related actions, processes and objects organized into a larger mental structure called schema (Confrey and Smith, 1994). It is actually a tool which researchers employ clarify how students develop or fail to develop, their appreciative of mathematical concepts. For instance, when given a task, an individual may accomplish the assignment correctly, another may have problems, and still another may fail completely. As observed by Dudinsky (2010), when structures involved in learning a certain concept are detailed, a genetic decomposition can aid an instructor to expose sources of challenge that arise in the learning procedure. The genetic decomposition may be used to explain variances in presentation (Strayer, 2010). The one who is successful may give confirmation of having successfully made one or more of mental formations entitled for by genetic decomposition. The student shows limited progress may show evidence of having begun to make constructions. The failed student may not have made the constructions at all or may give confirmation of having been unsuccessful in having made the required constructions (Arnon et al, 2013).

Therefore in this particular case, the genetic decomposition guides the analysis as well as pointing out on the openings in the researchers’ appreciative of how the concepts grows in the mind of the individual. Moreover, Dubinsky and Wilson (2013), suggest that genetic decomposition can also assist to lead the design of an instruction by providing an account of how a concept might develop in the world of a student. If the transformations in students’
performances cannot be enlightened by the genetic decomposition that would imply that the genetic decomposition needs alteration (Kirk, 2013).

2.4 STUDENT CONCEPTUAL UNDERSTANDING AND TEACHER’S CONTRIBUTION TO IT

There are various frameworks on how students learn mathematics. Jeffs and Smith (2018) argue that for a teacher to prepare teaching methodologies that promote, he or she should have an insight into factors that lead to a learner’s conceptual understanding and procedural understanding of a particular concept. Piaget cited in Chinamasa (2008) talks about conceptual and procedural learning of mathematical concepts. Procedural knowledge is chiefly concerned with the steps or processes followed as one learner learns a concept whereas conceptual knowledge is on the contrary it involves mastery of content and ability to transfer learnt knowledge in a different context easily. By emphasizing conceptual understanding, there is a chance of reconstructing a process that one may have forgotten. In other words, students have more to work with not just a procedure (Aroson and Patnoe, 2011). Therefore, it can be said that mathematical understanding basically exists as procedural knowledge, conceptual knowledge or both. Students can display conceptual understanding which can be viewed as integrated functional grasp of mathematical concepts. Students can also recognize why a mathematical concept is crucial and even the kind of circumstances in which it is useful. Additionally, they have information organization comprehensible making it possible to develop new ideas through linking their existing knowledge with the new information. This was supported by Strayer (2010), conceptual considerate also supports remembering of facts. Approaches that are learnt in appreciative way are very connected so that there are easy to recall and can be easily restored when
forgotten. When students are exposed to a technique and get to understand it, they are likely to remember it correctly when called upon to. When learners have conceptual understanding they can even explain a method to themselves and try to correct it when necessary (Nilson, 2010).

The learning process and the process of understanding are ongoing processes that lead to full understanding according to Engelbrecht, Harding and Potgieter (2010). They further suggest that the processes of understanding new mathematical concepts occur in levels in which with every level the student understands deeper. They claim that for learners to gain deeper understanding, they have to be repeatedly exposed to the intended concept. In addition, they suggest that mathematics learning consists of two processes which are first time exposure and consolidation. Real learning and understanding comes about by working on many exercises of a certain nature and this also brings recurrent experience and huge understanding. Relationships and connections imply that students possess information which they employ to make some sense of a problem in order to come up with new knowledge by making connections with the existing one. (Carpenter & Lehrer, 1999). Further, students start to create relationships when asked to solve problems that inspire them to use their informal mathematical ideas in combination with prior mathematical knowledge (Confrey & Smith, 1994).

Most of these studies indicate that teacher’s instructional practice is rely upon his or her knowledge (Brijlall and Maharaj, 2010). While secondary mathematics teachers’ content knowledge is usually judged as satisfactory for teaching, teaching for understanding needs more than simply well-developed content knowledge (Chazan, 1992). In addition, this body understanding of research has shown the relationship between teachers’ knowledge and their instructional practices is not as simple initially imagined. There is a difference between knowing and knowing how to use it in practice. Carpenter and Lehre (1999), claimed that
connecting teachers’ knowledge of mathematics to the knowledge on how students understand mathematical concepts is of utmost significance. When this is accomplished, then the teacher will be in the right position to choose appropriate pedagogical approach to deliver a given concept.

Arnon et al (2013) asserts that there is growing research backing for designing classroom instruction that concentrations on developing deep mathematical knowledge about mathematics processes. When instruction is concentrated only on skilful execution, students develop programmed procedural knowledge that is strongly connected to any conceptual knowledge network (Strayer, 2012). This instruction resulted in actions not implemented ‘intelligently’ and systematically.

Understanding could be attained, however, if students were provided with more opportunities to cultivate a framework for understanding suitable relationships, extended and applied what they knew, shown on their practices, and made mathematical knowledge of their own (Carpenter & Lehrer, 1999). They further assert that (a) when mathematical knowledge is understood, that knowledge is more easily recalled and more readily useful in a variety of situations, (b) when a unit of knowledge is part of a well-linked system of mathematics APOS theory used in Brijlaal and Maharaj (2010) distinguished the hypothesis on learning, that an individual does not learn mathematical concepts directly.

This is drawn from Dubinsky (2010) who asserted that the individual uses mental organisations to make sense of a concept. Brijlall and Maharaj (2010) claims that learning is enabled if the individual retains mental structures applicable for a given mathematical concept, and that if the mental structures are not there, then learning becomes almost unmanageable. It is therefore crucial for the teacher to improve teaching strategies that assist students to improve their conceptual understanding of mathematical concepts.
2.5 SUMMARY

In this chapter, a theoretical framework of the APOS theory which guides the study was discussed. Also literature on group concepts was discussed. The researcher also discussed literature on students’ mental constructions and genetic decomposition of group concepts and factors that lead to achievement of students’ conceptual understanding of mathematical group concepts. In chapter three research methodology used to analyse the student’s conceptual understanding of group concepts will be discussed. In chapter four the researcher will present and analyse data for the study and lastly in chapter five the summary, conclusions and recommendations of the study will be highlighted.
CHAPTER THREE

Research methodology

3.0 Introduction

The chapter describes the research design employed in the study. Participants, research instruments, data collection procedure as well as data presentation and analysis procedures are going to be outlined.

3.1 Research Design

In this research a case study research design was used to analyse student’s conceptual understanding of advanced level group concepts. This design was selected because it allows the researcher to study the participants in their real setting (Yin, 1994). Also the design was selected since it allows the researcher to select a small geographical area which is the school in which the study was conducted and a limited number of students as subjects of the study so as to get detailed contextual analysis of students understanding of group concepts.

In this research, a single case incorporated with subunits was used (Yin, 1994). This is refers to the Advanced level class with ten students where the study is to be done on each and every student in the class. The study will seek to analyse students’ conceptual understanding of group concepts. After four lessons an in-class was given in order to implement the APOS theory objectively so as to explain the students’ difficulties or challenges in conceptual understanding of group concepts. Follow up interviews were conducted on students so that the researcher can identify the stage of each student according to APOS theory.
3.2 Research participants

The research participants were made up of the advanced level mathematic students in form six at Mutondwe High school in Mt Darwin District. The class has a total of ten student, eight males and two female students. This clearly reflects the notion that mathematics is a masculine subject which is done by males. The student’s ordinary level performance in mathematics is outlined in the table below

**Ordinary level results for participants**

<table>
<thead>
<tr>
<th>Student</th>
<th>Gender</th>
<th>Ordinary level results</th>
</tr>
</thead>
<tbody>
<tr>
<td>student 1</td>
<td>male</td>
<td>A</td>
</tr>
<tr>
<td>student 2</td>
<td>female</td>
<td>B</td>
</tr>
<tr>
<td>student 3</td>
<td>male</td>
<td>C</td>
</tr>
<tr>
<td>student 4</td>
<td>male</td>
<td>B</td>
</tr>
<tr>
<td>student 5</td>
<td>female</td>
<td>B</td>
</tr>
<tr>
<td>student 6</td>
<td>male</td>
<td>B</td>
</tr>
<tr>
<td>student 7</td>
<td>male</td>
<td>A</td>
</tr>
<tr>
<td>student 8</td>
<td>male</td>
<td>A</td>
</tr>
<tr>
<td>student 9</td>
<td>male</td>
<td>A</td>
</tr>
<tr>
<td>student 10</td>
<td>male</td>
<td>A</td>
</tr>
</tbody>
</table>

Table 3.1 *Ordinary level results for the participants*

The students performed well in their ordinary level mathematics examinations.

3.3 Sampling size

A sample of ten Advanced level Mathematics learners at Mutondwe High School in Mt Darwin District in Mashonaland Province was purposively sampled to provide information for the study. The researcher utilised purposive sampling to select the participants for the study as it allows the researcher to include every student in the Advanced level mathematics class.
3.4 Research instruments

The study used an in-class test and interview guide to solicit data from respondents. The in-class tests were the instruments used to gather primary data. To get clarifications on some unclear circumstances, interviews were used. They enable some issues which could not be addressed well in the post-tests to be clarified and attended to.

**In-class test**

The in-class test by plan covers all the topics which a student will have been taught, (Efrat and Ravid, 2013). It was designed to measure the amount of learning a student has assimilated in a specific subject, in this case Group theory. To do this, questions regarding all the concepts covered in Group theory must be included in the test. Numerical score is assigned to the test so as to measure the progress made during the learning process. In this study an in-class on Group theory concepts was given. (see appendix).

According to Rosenbaum (2010), a post-test functions as a teacher-diagnostic tool in different ways:

- It identifies concepts which students have not mastered well
- It identifies concepts which students have mastered very well

The in-class test, to a teacher, is very crucial in that weak students and/or strong students are identified. It is the teacher’s obligation to offer more remedial instruction to weak students. It is also the teacher’s onus to give additional challenging material to strong students.

It is the duty of the teacher to prepare the teaching programme of the class. The results of the post-test will provide the instructor on what to adjust on the scheme of work. According to Stephen (2013), if the post-test for a previous lessons indicated that most students did not
perform very well, a wise teacher would revise his teaching tactics and perhaps use different teaching resources for the next lessons. This therefore implies that post-tests are an essential teacher-diagnostic tool for measuring the learning of mathematics concepts like Group theory so that teaching can be more effective.

**Interview Guide**

Interviewing, according to Rosenbaum (2010) is typically done face to face. In this study, the interviewer asked respondents a set of questions specifically designed to get answers for the research questions.

Coglan and Brydon-Miller (2014) observe that an entirely structured interview is basically a questionnaire administered by an interviewer who is not allowed to depart in any way from the questions given. The interviewer merely reads out the questions to the respondents. On the other hand, an absolutely unstructured interview takes a arrangement of a conversation where the interviewer has no predetermined question (McNiff, 2014). It would appear as if most interviews fall somewhere between structured and unstructured. In this study the researcher used a structured interview. (see appendix)

This technique of data gathering requires the researcher to prepare questions in advance so as not to diverge from the required research but need also to be flexible to allow new leads which may not have envisaged. The interviews facilitated a personal contact between researcher and respondents. Probes were made in a face to face situation. This helped the researcher to gather all the important information. Leading questions to such as emotions and gestures were observed or felt. On the hand, Kirk(2013) noted that interviews have limited number of subjects in a sample due to one procedure. Leedy and Ormrod(2013) also observed that in the presence of a researcher may cause bias to the responses.
The researcher arranged appointments and visited interviewees at their work places so as to overcome low turn up. The chances of bias were reduced by designing a structured interview schedule. The wording and sequence of the structured interview schedule were fixed and identified for every respondent. The main intention of these characteristics was to ensure that when differences developed between answers, it would be attributed to the actual variation between respondents and not the differences in the interview.

Interviews contribute a lot in qualitative research in that they enabled to collect details about peoples’ beliefs, feelings, motives, past and present behaviour and events. Interviews in qualitative research, offer an advantage of flexibility in that they are mostly unstructured or semi-structured. Interview can also take the form of a focussed group discussion and it has an advantage on saving limited time.

Interviews as research instruments offer a good number of merits. The researcher agrees that the interview technique is likely to indicate more information since the researcher can gain access to what is “inside a person’s head.” During an interview the interviewer can also show flexibility and can repeat or restate questions so that the respondents comprehend what it meant by a particular question.

More so, people are willing to talk than writing. In this study, the interview seemed to be the appropriate tactic for gathering information about complex, emotionally loaded areas or probing the situations that may underline an expressed opinion. Paul et al (2012) argue that an interview allows for much greater depth than the other methods of collecting research data.

However, interviews as a research tool have some demerits. This research tool is time consuming and is one of the most difficult to use successfully. In an interview, bias is always
encountered. Bias invalidated research study findings. Interviewing, also affects the recording of observations and the presence of the interviewer may influence the respondents’ thinking. Despite the demerits mentioned, the interview technique remained the most appropriate instrument of data collection to complement quantitative data in this study.

3.5 Data Collection Procedure

The researcher obtained permission from Mutondwe High School administration to conduct the research after producing a letter from Bindura University of Science Education to carry out a research. The teacher conducted four lessons on group concepts to Advanced level students. Later, an in-class on covered concepts on group theory was administered. The test was marked and recorded of individuals’ programmes. Face to face interviews were to be done at respective classrooms of the students. For this study, judgemental sampling was used. An introductory letter from Bindura University of Science Education (BUSE) was produced so that the respondents would be assured that the study was to be used for academic purposes only.

3.6 Data presentation and analysis procedures

Information presentation is an central part of any research development and without an appropriate analytical procedure, it is not proper to come up with meaningful conclusions (Creswell and Creswell, 2018). The study results were to derived from in-class test given to the maths Advanced level class and also from follow–up interview to the students. The data from the in-class test analysed through tables and extracts that indicated the performance of selected students. The information from interview guide was presented by way of interview, extracts where interviews conducted were taken as they were.
3.7 Summary

This chapter outlined the research methodology used in the study. The single case study research design with multiple units was used. An Advanced level class was used to analyse students’ conceptual understanding of group concepts using APOS theory. In-class test and interview guides were the main data collecting instruments. In the following chapter data presentation and interpretation of findings is going to be presented and lastly in chapter five the summary, conclusions and recommendations of the study will be given.
CHAPTER FOUR

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.0 INTRODUCTION

In the chapter, the researcher is going to dwell on the presentation and analysis of data collected. Data collection tools that were used included questionnaires, follow-up interviews and tests which were all designed in such a way that an in-depth analysis of students’ conceptual understanding of group concepts at Advanced Level could be done holistically. Excerpts was used to present data collected by interviews. Selected learners’ work and follow-up interviews were presented and analysed. A qualitative analytical approach was predominantly used in this research. The selected participants were asked various questions as a follow up of the responses they supplied in the test they wrote. All this was aimed at providing answers to the research questions.

4.1 CLASSIFICATION OF ADVANCED LEVEL STUDENTS ACCORDING TO APOS THEORY.

The researcher is going to use the written tests and interviews to demonstrate and provide evidence of the different APOS Levels of conceptual understanding learners operate at in learning group concepts. The preliminary genetic decomposition accounted for in the previous chapter will form the basis of the analysis of learner conceptual understanding of groups.
4.1.1 Results from the in-class test.

<table>
<thead>
<tr>
<th>Student</th>
<th>Mark (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>student 1</td>
<td>76</td>
</tr>
<tr>
<td>student 2</td>
<td>63</td>
</tr>
<tr>
<td>student 3</td>
<td>73</td>
</tr>
<tr>
<td>student 4</td>
<td>67</td>
</tr>
<tr>
<td>student 5</td>
<td>63</td>
</tr>
<tr>
<td>student 6</td>
<td>80</td>
</tr>
<tr>
<td>student 7</td>
<td>57</td>
</tr>
<tr>
<td>student 8</td>
<td>60</td>
</tr>
<tr>
<td>student 9</td>
<td>57</td>
</tr>
<tr>
<td>student 10</td>
<td>42</td>
</tr>
</tbody>
</table>

Table 4.1 Students’ results from the in-class test

The statistics showed that the students performed satisfactorily from the in-class given. Student 6 scored 80% which is a sign of having understood the concepts on group theory. On the other hand, student 10 achieved 42% which indicated that there some areas where the student is in need of assistance. However, overall the performance was pleasing since a 90% of the students scored above half.

4.1.2 Nature of test items

The test instrument used consisted of two sections. Section A consisted of four questions where first question required the definition of a group, second question tested learners about their mastery of basic group properties, third and fourth questions sought to elicit learners preliminary understanding of binary operations as they form the basis of group concepts. Then, section B comprised questions on the formulating groups using the method of constructing tables and finding the order of a group by counting elements in the table, however this section was divided into two sub-questions where first part led to the construction of the tables and the second part only required the counting of elements to find the order of a group and use the table to find some elements for specified operations. The
main idea behind all this was to try and simplify the complexities that are usually associated with most group questions.

### 4.1.3 Understanding of the definition of groups.

As already alluded to, question 1 was concerned with defining a group. Learners gave a variety of written responses and their responses were categorized according to key elements in their definitions. See the table with such information and the frequencies for each category.

<table>
<thead>
<tr>
<th>Category</th>
<th>Definition (indicator)</th>
<th>Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Is a set.</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Is an ordered pair.</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Is asset of elements</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Totally irrelevant definitions.</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>correct definition</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 4.2 Frequencies of scores for definitions of groups.**

These statistics showed that 3 students understood a group as a set, 4 students consider a group to be an ordered pair, 2 students linked a group to a set of elements. Their thinking is in line with the first 3 students. Those who gave totally irrelevant definitions were 3. These did not have a clue of what a group is. The students who managed to define a group were 3 which according to the preliminary genetic decomposition, suggested that they are operating at an Action level of the APOS Categorisation format. It can be noted that the majority of the students in class were still struggling to conceptualize the definition of a group. The
following were some of the selected learners’ responses to question 1 and subsequent interviews:

**Student 1’s definition of a groups.**

She wrote: a group is a set of elements like those of a set. The student also failed to state the properties of a group

Analysing her response it can be seen that Student 1 roughly understood the group as a set of elements. The student is trying to associate the concept of a group to the concept of sets

<table>
<thead>
<tr>
<th>Interviewer: What makes you say it is a set?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1 : we have elements in it.</td>
</tr>
</tbody>
</table>

Interviewer: So all those things that put elements together are sets?

| Student 1 : Yes. |

Interviewer: Can you identify any property of the group in what you said

| Student 1 : identity element. |

Interviewer : What is an identity?

| Student 1 : It is an element of a group. |

Interviewer: Is that all?

| Student 1 : Yes. |

---

**Figure 4.1:** Student 1’s interview excerpt for question 1.
The follow-up interview reflected that student 1 indeed was operating at the action stage of the APOS. She could simply regurgitate what she had learnt without being able to think beyond that.

Student 2’s understands of the definition of a group

Student 2 wrote: A group is a set of ordered pairs.

Student 2 generally understands or remembers that a group is an ordered pair but is not clear on the difference between a group and a set, however a follow–up interview was conducted to ascertain whether or not the student really understands the concepts of definitions that he gave. The following is an excerpt of the interview that was conducted between him and the researcher.

<table>
<thead>
<tr>
<th>Interviewer: Are you saying that it is an ordered pair?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 2 : Yes,</td>
</tr>
</tbody>
</table>

Interviewer: Okay. So how is it similar to a set?

| Student 2 : in a group we have elements and in a set we also have elements |

Interviewer: Good. From your answer it seems there is a link between a group and a set.

| Student 2 : Yes. |

Interviewer: So can we say they are the same?

| Student 2 : Yes, both can be represented by listing elements. |
Figure 4.2: Student 2’s interview excerpt question 1.

Student 2’s response showed that the student had a different conception of the definition though he seemed to be at the same level of operation with student 1. His conceptual understanding of the definition was a bit deeper as he brought about the concept of an ordered pair in the definition and he demonstrated good confidence which enabled to think critically and even ask his own questions to the interviewer. With a bit of some clarifications from the teacher the student could operate at a higher level of understanding according to APOS framework.

Student 3’s understanding of the definition of a group.

Student 3 wrote: A group is an ordered pair and it satisfy some properties called the group properties.

Student 3 written responses showed that he had a considerably deeper understanding of groups. His description was full of clarity and detail. His writing further confirms that he actually go an extra mile in school work, he does not solely rely on what the teacher says in the classroom. The following is an interview that was held between Student 3 and the interviewer.

| Interviewer: Student tell me what you meant by an ordered pair. |
| Student 3 : It is a binary |
| Interviewer: Is that what you mean by ordered pair? |
Student 3: I can give an example.

Interviewer: go ahead

Student 3: let me try but I am not sure.

Interviewer: You can go ahead and try.

Student 3: it must satisfy some properties

Interviewer: You cannot be more right, wonderful. Is there anything you want me to explain to you on the properties?

**Figure 4.3: Student 3 interview on question 1**

Student 3’s oral response showed that he really understood the definition of a group. The student’s level of operation can be placed far above Action level of the APOS as confirmed by his oral responses. He can naturally fit in this category according to the preliminary genetic decomposition of group concept.

**Student 4’s understanding of the definition of group.**

Student 4 wrote: A group is a group of things.

Student 4 is clueless about the definition of group even though he could highlight in the definition the term group and try to give some properties though they were wrong. This indicates that Student 4 was operating below the Action level of the APOS. Below is an interview which further made it clear that Student 4 was operating short of the APOS threshold (Dubinsky, 2011).
Interviewer: you are simply mentioning group in your definition?

Student 4: I can’t remember it very well

Interviewer: So you mean you do not have any clue

Student 4: No, there are some properties but I have forgotten their names.

Interviewer: Then, why did you not write that in your written response?

Student 4: It just did not come into my mind, and I only remembered the word group.

Figure 4.4: Student 4’s interview excerpt question 1

4.2 CONCEPTUAL UNDERSTANDING OF A SUB-CONCEPT OF SOLUTION OF GROUPS.

The researcher will analyse and discuss participants` written and oral responses on solution of groups as this will lead to solving of group problems. According to preliminary decomposition of the concept of groups, mastering this sub-concept is an indicator of a student being at process level of the APOS theory. An analysis of selected works by the participants.
Student 5’s understanding of solving binary operations.

Figure 4.5: Student 5’s test responses- Section A.

Student 5 showed that she lacked basic knowledge on how to manipulate the tables to find elements of the group with the defined binary operation, her procedure of solving both simple binary operations and group questions was quite incorrect. As such, she ended up getting wrong answers owing to her lack of deep understanding of binary operations. She also had little knowledge about formulating tables to represent binary operations when dealing with situations where the tables are required. Considering her solution, Student 5 was operating below the process stage of the APOS.

The subsequent interview also reviewed that Student 5 was indeed operating below the process level. Below is the excerpt from her interview.

<table>
<thead>
<tr>
<th>Interviewer: Why are you not using a table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 5: umm mi can’t make it.</td>
</tr>
</tbody>
</table>
Interviewer: Is that so?

Student 5: sir. can you assist

**Figure 4.6: Student 5’s interview on question 3**

Student 5’s interview response was consistent with the written response she gave in the test and this therefore indicated that she was operating at lowest level of understanding.

**Student 6’s understanding of solving binary operations.**

![Image of handwritten notes showing binary operations](image)

**Figure 4.7: Student 6’s test responses–Section A**

Student 6 could solve question 2 which was on binary operations. He demonstrated a deep understanding of groups and binary operations as he could draw tables to represent them. He however, had a difficulty in stating the identity element of the group from the table. He showed that he was clueless with respect to the aspect of using the table to answer further questions. The fact that he could answer question 4 correctly helps to place him at process
stage of the APOS since he could very well follow the procedures as they were taught in class. In the follow up interview, Student 6 indicated that he does not know what an identity element is. Further probing could not help him to find out the identity from the table.

**Student 7’s understanding of solving of binary operations.**

![Figure 4.8: Student 7’s test responses-section A](image)

Student 7 could illustrate the binary operation with a table. That ability prepared a smooth transition from giving definition of groups and binary operations to solving specific questions of binary operations such as finding an identity element and inverse element. Student 7 managed to solve, questions numbers 3 and 4 correctly. He demonstrated both conceptual and procedural understanding of solving group and binary operations. This was further substantiated by his inability to give correct elements of identity element and inverse element in the follow-up interview. The following is the excerpt from Job’s interview.
Interviewer: tell me, what is a identity element?

Student 7: It is an element that when operated with another element we get the same element.

Interviewer: Quite correct. In this table what is the element?

Job: for this binary operation it is the element zero.

**Figure 4.9: Student 7’s interview excerpt-question 2.**

From his test responses and interview responses, Job was operating at process stage of the APOS theory.

**Student 8’s understanding of solving binary operations.**

![Image of a table]

**Figure 4.10: Student 8’s test responses-section A.**

Student 8 showed that she has a high level of both conceptual and procedural understanding groups and binary operations. She could answer questions 1 and 2 on defining groups and stating properties of groups and went on to answer questions 3 and 4 very well. She even reviewed that she has an in depth understanding of groups and binary operations. She constructed all the tables very well for representing binary operations and managed to identify the identity elements in all the tables. The following is the interview excerpt of Student 8
Interviewer: I can see you could solve questions 1 up to 4 very well, is there a question you found challenging?

Student 8: I cannot quite say there was a difficult question for me, but I had a challenge on time management. I did not finish my work. The time given was not enough for me.

Interviewer: So asked to give inverse element for question 3, can you do that?

Student 8: Certainly, sir.

Interviewer: Then, go ahead.

Student 8: 1…

**Figure 4.11: Student 8’s interview excerpt –SECTION A**

Considering Student 8’s responses in totality, there is sufficient evidence that she was operating at a level above the Process stage of the APOS theory. All this is based on the preliminary genetic decomposition of groups described in chapter 3.

**4.3 CONCEPTUAL UNDERSTANDING OF GROUPS AND BINARY OPERATIONS.**

The participants answered a question in section B which was on construction of tables. They demonstrated different levels of abilities. An analysis of four representative works selected was done.

**Student 9’s understanding of concept of construction of tables for groups.**

The student could draw the tables as was required. She was also capable of inserting all elements according to the binary operations described in the. However, she could not come
up with the required elements for further questions concerning the table. In other words she could not use the table to answer the required questions.

The following was her follow-up interview after the test was marked.

**Figure 4.12: Student 9’s interview excerpt on question 5a**
Both the interview and the written responses indicated that Student 9 was operating below the Object stage of the APOS theory, thus, according to the preliminary genetic decomposition.

**Student 1’s understanding of graphing of construction of tables for groups**

Student 1 managed to draw all the tables for the two binary operations one for addition and other for multiplication. She also managed to find the identity elements in both operations. The subsequent revealed that Student 10 is able to construct the tables and has conceptualised...
the technique. Figures 4.12 show Student 10’s written responses to question 5a and the follow-up interview excerpt on question 5a respectively.

![Figure 4.14: Student 1’s written response to 5a](image)

**Interviewer:** I can see in your written test, you manage to draw the tables correctly, how did you manage?

**Student 1:** No, sir. I did not use that method.

**Interviewer:** What method did you use, then?

**Student 1:** Sir, I remember very well how it was done in clas so i followed that.

**Figure 4.15: Student 1’s interview excerpt on question 5a**
It can be established that the student was operating at the process level of the APOS. She could draw the tables well but may not qualify to fall in the category of the object level of the APOS theory as provided by the preliminary genetic decomposition. The reason was that the student even though the tables were accurately drawn, she could not successfully complete entering all the information that is associated all the tables.

**Student 9's conceptual understanding of construction of tables for groups.**

Student 9 managed to draw the tables of all one binary operation for addition and did not complete the one for multiplication. He could demonstrate that he understood the link between different elements as described by the binary operation.

![Figure 4.16: Student 9's response to question 5a](image)

Even though the student could draw the correct table for the binary operation on addition, he could not complete the table for multiplication. The subsequent interview with him revealed that he was really devoid of how tables for multiplication are constructed. He took it for granted that the given statements will automatically be used to the same operations. The following is the interview excerpt on question 5a with Student 9.
Interviewer: why have not completed the other table?

Student 9: it is difficult for me.

Interviewer: Do you remember of anywhere you have seen such a table or in a textbook.

Student 9: I have not noticed the difference,

Interviewer: multiplication is different from addition operation.

Student 9: Oh, I see.

Figure 4.16: Student 9’s interview excerpt on question 5a.

With the above responses from the student, it can be established that the student was operating at process stage and so failed to meet the full criteria of the preliminary genetic decomposition consistent with the Object stage of the APOS theory.

Student 7’s understanding of concept of construction of tables for groups.

Figure 4.17: Student 7’s response to question 5a.
In figure 14.15 above, the student was able to draw all the tables for the given operations. His ability could be ascribed to the fact that he managed to define the groups and give all the properties of the groups in the previous questions. He also could identify identity element and the inverse element for each operation correctly. However, the student was not able to answer further questions concerning the operations. This omission disabled the student to fit at Object. Stage of the APOS since the preliminary genetic decomposition stated that one has to be able to do the tables part successfully for him or her to be said to be operating at the object stage. In the follow-up interview, the student proved that he indeed did not know how to respond to further questions. Therefore the student was operating at the process stage of the APOS.

4.4 Conceptual understanding of binary operations.

Question 5a (ii) required that the participants demonstrate to apply their abilities to do programming aspect of linear programming by means of minimising the given objective function. And the last question (question 5b) sought the students’ abilities to formulate constraints or inequalities from the given word problem. Their responses were marked and follow-up interviews were conducted in order ascertain the APOS levels of four selected representative participants. The following is analysis of the four selected participants.
Student 4’s understanding of binary operations.

![Image of student's work]

**Figure 4.18: Student 4’s responses to 5a (ii) and 5b.**

From the student’s work shown in figure 4.15, it can be seen that the student was virtually clueless in terms of the required responses to these two questions. He only gave and was way far away from the expected answer. In short, he did not understand the questions at all. The follow-up interviews indicated that student could not formulate inequalities from word problems. The following is the interview excerpt with student 4.

**Interviewer:** can you clarify your point to me?

**Student 4:** Yes, sir I will try.

**Interviewer:** What was so difficult?


Student 4: I cannot find the inverse using the table

Interviewer: try to remember what you did in class?

Student 4: I did not understand.

Figure 4.19: Student 4 interview excerpt on question 5a (ii) and 5b.

The above interview in figure 4.16, The student demonstrated lack of conceptual knowledge on using the tables.

Student 8’s understanding of group operations.

The student though he could illustrate the operations on the table correctly, the mistakes or omissions he made contributed to his subsequent failure to fully understand the requirements of the last questions. He could not even attempt to find the solutions of other operations using the table because of its incompleteness. He lacked interest and practice as was pointed out by the follow up-interview which was conducted following the written test. He could express some of the procedures required to get elements of the group that can lead to finding the inverse of the group or the identity of the group. Surprisingly though, he could write anything that was suggestive of the knowledge he displayed in the follow up-interview. Considering what he wrote and what he said in the follow up-interview, the student was actually operating at the Process level of the APOS. This determination so, because whilst he could perform well in the test, it could be concluded that he according to Dubinsky (2001), he possesses the mental structures that can enable him to perform the procedural processes of solving the problem.
All that he lacked was the deep understanding of the concept under discussion and to some extent he also lacked interest in tackling more difficult questions. The following figure 4.20 show the student’s follow up-interview responses respectively to questions 5a (ii) and 5b

<table>
<thead>
<tr>
<th>Interviewer: Can you tell me the meaning of the operation * in your table.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 8: the operation represent addition</td>
</tr>
<tr>
<td>Interviewer: Correct, but you did not show that in the test.</td>
</tr>
<tr>
<td>Student 8: I was confused in the test.</td>
</tr>
<tr>
<td>Interviewer: what do think could have been the cause of your confusion if you could answer the question like you have done now?</td>
</tr>
<tr>
<td>Student 8: I Think it is lack of practice.</td>
</tr>
</tbody>
</table>

**Figure 4.20: Student 8’s interview excerpt on question 5a (ii) and 5b.**

From the follow up-interview in figure 4.20, lack of practice and interest militate against student8’s level of conceptual understanding of group tables.
Student 7’s understanding of binary operations.

Figure 4.21: Student 7’s written response to questions 5a (ii) and 5b.

From figure 4.21, the student could draw the tables to represent operations of the group. A follow up- indicated she was good with language as she could answer questions to do with keyword-symbol correspondence doing operations. In the follow-up interview, she indicted that she could not write the the identity and inverse elements due the little time she devoted that part, and she could provide the third interview in the follow-up interview. Student according to the preliminary genetic decomposition, was operating at object stage since she could demonstrated procedural understanding and a bit of deep conceptual understanding. She could not progress to schema stage since she could not tackle problem solving questions of higher order, in this case she was not capable of solving a question on groups.
4.5 Overall distribution of the students on the APOS scale.

<table>
<thead>
<tr>
<th>APOS LEVEL</th>
<th>LEARNERS</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACTION</td>
<td>Student 3, 2, 5, 4, 1, 6, 7, 9</td>
<td>8</td>
</tr>
<tr>
<td>PROCESS</td>
<td>Student 1, 4, 6, 5</td>
<td>4</td>
</tr>
<tr>
<td>OBJECT</td>
<td>Student 7, 9, 8</td>
<td>3</td>
</tr>
<tr>
<td>SCHEME</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.3: Distribution of students’ mean conceptual understanding on APOS theory.

From table 4.3, the majority of the learners fell in the lowest level of the APOS which is the Action level. This indicated most students could provide definitions and perform very simple problems like stating the properties of a group. Most of them hardly draw the group tables completely. This could be attributed to limited time that was afforded to the learners to interact with the learning material Chinamasa et al (2014). The learner low conceptual understanding can be ascribed to learner attitude and misconceptions. Certain pupils would come for the lessons even without necessary learning equipment like pencils. It can also be noted that only four students were operating at the Process stage of the APOS theory. These students were able to use formulae or learnt procedures to tackle explicit problems.

Finally, only three students were operating at the Object stage which is the penultimate stage of the APOS theory. These students were able to perform high order questions like finding inverse element and solving problems which involved more operations. These students a lot of enthusiasm in the concept and they are ones who could do further consultations with various teachers. These students had most of the learning material necessary for learning—they displayed due readiness to learn. However, no student could be seen operating at the Schema
level of the APOS theory. This means no student was able to convincingly transfer or apply the learnt concepts into solving virtually new scenarios. This could have been due to lack of adequate practice on the part of the learners. Availability of more time for teaching and learning of this concept could have yielded better results.

4.6 SUMMARY.

In this chapter, the researcher gave explanations of how students in Advanced level maths class constructed concepts of groups. Analysis which was presented in tables and learners` written work extracts helped to explore the conceptual understanding of these concepts using APOS theory and the levels on which mental constructs on group concept were made using the preliminary genetic decomposition. Follow-up interviews were used to check on the answers on the written scripts. The researcher will conclude the study by discussing the findings, thoughts and recommendations in the next chapter.
CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS.

5.0 INTRODUCTION

In this chapter, the researcher will discuss the findings of the study which sought to establish the students’ conceptual understanding of groups at Advanced level in Mt Darwin District. The main objective of this research was to analyze students’ conceptual understanding of groups based on the theory of APOS with the intention to find means and ways to help improve teaching and learning of this concept. The chapter is going to focus on the conclusive review of the research findings then give some possible recommendations according to the results obtained in the investigation. In the next sections, I will discuss the students’ conceptual understanding levels and their implications.

5.1 Summary of the project.

The aim of the research project was to explore the students’ conceptual understanding of groups at advanced level. The study was a case study of an Advanced level maths class at a secondary school in Mt Darwin district. In chapter one, the researcher dealt with the background of the study which pointed out the crucial role mathematics plays in the general life of students. It also highlighted the importance of conceptual understanding when students learn group concepts and mathematics. The research questions, statement of the problem, definition of key terms, limitations and delimitations of the study were also dealt with. Then chapter two dealt with the review of related literature to the research problems. It reviewed literature on an analysis of students’ conceptual understanding of Advanced level group concepts. Also literature on group concepts was discussed. The researcher also discussed on
literature on students’ mental constructions and genetic decomposition of group concepts and factors that lead to achievement of students’ conceptual understanding of mathematical group concepts. The researcher went onto chapter three where he looked at research methodology: sampling techniques, data collection instruments that comprised test instrument on groups, follow-up interviews and questionnaires and issues to do with research ethics.

In chapter four, the researcher dealt with the presentation, analysis and discussion of data collected. The students’ conceptual understanding was determined using the preliminary genetic decomposition of the APOS. Students’ written responses were extracted and illustrated, then discussed in a bid to make due analysis. A follow-up interview excerpts were presented to help ascertain learners’ conceptual understanding suggested by written responses. The results obtained from the written responses and subsequent interviews pointed to the fact that learners operate at various conceptual levels as they learn group concept. The reasons behind these differences ranged from learner and teacher attitude, availability of teaching and learning resources and teaching approaches.

The data that was collected was illustrated in the form of tables which were analysed and compared. Students’ conceptual understanding of groups reflected in the tests and follow-up interviews was decided basing on the initial genetic decomposition, but it was discovered that some of the responses given by the students displayed mental constructs that did not link to the preliminary genetic decomposition in chapter one.

5.2 Conclusions

This section has provided answers to the research questions. Results reveal that the students operate at different levels of the APOS depending on the nature of the concept being studied. Generally, most students operated at action level of the APOS. Very few interiorised the action level to the process stage. Only three students managed to go through encapsulation of
process stage to the object level. No learner could reach the schema level of the APOS on the concept of groups.

5.2.1 Advanced level students `conceptual understanding of groups.

The main aspects that were considered in the written tests and interviews included: ability to define groups, state properties of a group, construct tables to represent groups, and finding the inverse and identity element of a group. It was observed that all students attempted to the question on definition of groups but their definitions were incomplete. On the second part, about four students were unable to solve the groups problems totally indicating that they were operating at pre-action stage of the APOS. Most students were unable to state the identity and inverse element of the group.

On construction of tables question, not all students managed to draw the tables correctly, hence not all students reached the action stage. The interiorisation of action stage into process stage was to be indicated by the ability draw the complete table with all the elements of the group. However, the process stage was to be encapsulated into object level by the ability to connect operations of a group to the elements in the table. This implies that no student could progress to object stage on this aspect. The last aspect concerned solving other related questions using the table they have constructed. Most students showed no clue about what was required at this stage. Only three students could attempt to answer the questions but the answers that they gave were not convincing of having high conceptual understanding. Therefore majority of the students operated at action stage.
5.2.2 The extent to which the preliminary genetic decomposition explained the advanced level students’ responses.

It was noted from the data analysis that the preliminary genetic decomposition could not account for all students’ responses. Some responses were not clearly written, hence there was need for a follow-up interview to probe and understand the students` mental constructions based on the written responses.

5.2.3 Students` weaknesses on the concepts of groups.

It was observed that most students had problems with the use of English. They could not interpret what the given word problems mean and they could not understand the requirements of the questions as the answers that they gave were divorced from the expected answer. Furthermore some students which form the majority could not attempt word problems questions. Secondly, some students had the weakness of using the given group operations.

5.3 Recommendations

Affording advanced level students more time to study groups.

The fact that this study revealed that Advanced level students operate at different levels of understanding of concepts, it is therefore necessary to cater for these human differences by allocating more time to difficult concepts like groups on the school time table. It is also important to teach student teachers concepts that are difficult at advanced level during teacher training program as some teacher too face some challenges in understanding these concepts. Realising this may allow some mental constructions to be pushed to a higher level as the students interact more with some content. Action level maybe interiorised into process level and the process level maybe encapsulated into object level.
Use of instructional methodologies which foster understanding in class.

The use of ACE in this research was fruitful option, considering the available time, because the researcher noticed that the students interacted with each other in the classroom discussions, slow learners benefited a lot from the fast learners. The discussions helped some students to interiorise their mental constructions from action level to process level and those at process level to go through encapsulation in order to reach the object level. Teachers are therefore recommended to design instructional methodologies that help students to improve their level of understanding of group concept.

Misconceptions held by learners.

It was found that learners make a lot of misconceptions as they learn concepts. It is therefore necessary for teachers to identify these misconceptions and find ways to deal with them. Modern technologies (ICT tools) can be effectively used in the classroom to reduce these misconceptions. Teachers can share these misconceptions when they meet at district and provincial subject workshops and help each other on possible ways to eradicate the misconceptions held by learners.

Relating Mathematics concepts to real world situations.

ACE teaching methodology involved activities carried outside the classroom, giving the learners the chance to relate mathematics concepts to the real world. It was observed that most students appreciated the importance of mathematics, hence boosting their attitudes towards the subject. From this background, it can be recommended that teachers in schools can improve pass rates at final examinations by incorporating the ACE approach in their teaching rather than using expository strategy. This fosters the interest of the subject in mathematics learners leading to improved understanding of the concepts.
Effectiveness of exercises and homework

The exercises and homework given to the students proved to have reinforced the learners’ conceptual understanding of the concept taught. Students had the opportunity to build on their mental constructions as they kept on practising the learnt concepts. Teachers are therefore recommended to use homework and exercises effectively as this may help boost students’ level of conceptual understanding.

5.4 Summary.

In this chapter, the researcher gave brief details about the summary of the essential components of the project, the conclusions drawn based on the employed preliminary genetic decomposition of the concept of groups and APOS in general, and as well as possible recommendations emanating from the research findings. The researcher observed that the students operated at different levels of the APOS depending on the nature of the concept being taught. From the research on group theory most students operated at the action level of the APOS framework. It is therefore important to afford Advance level students more time to study and more exercises and home works should be given so to provide them with more practice.


Appendices

Upper Six Test

**TOPIC: Group Theory**

1) State five properties of binary operation. [5]

2) Let * be a binary operation defined on set A={1,2,3} by table below

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

a) Use the table to compute 1*3, 3*3, ((1*2)*3)*2. [1,1,1]
b) Is * commutative? Why? [1,1]

3)(a) Show that the set of integers {1,3,5,7} under multiplication modulus of 8 forms a group. [6]

(b) Decide whether (Z, *) is a group where * is defined by n*m=n+m-1. [5]

(c) Construct group table for the group (Z\textsubscript{7}, +\textsubscript{7}) and verify that it is commutative. [4]

(4) Show that (Q, ×) is not a group. [5]
Interview Guide

**TOPIC: Group theory**

1. Define a binary operation.

2. How do you perform this operation $1 \times 3$, from the given table?

3. How do you draw a table of integers $\{1, 3, 5, 7\}$ under multiplication modulus of 8?

4. How do you verify that $(\mathbb{Q}, \times)$ is not a group?

5. Can you tell me the meaning of $\ast$ in your table.