

NUMERICAL METHODS IN FINANCE

Time : 3 hours

-- DEC 2019

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. Distinguish between the following terms as they apply to numerical methods

- (a) Numerical Stability and Instability. [3]
- (b) Relative Error and Absolute Error. [3]
- (c) Continuity and Convergence. [3]
- (d) Implicit and Explicit Finite Difference Methods. [3]

- A2.** (a) Establish the Secant method for solving a non-linear equation. [6]
- (b) State two weaknesses of the Newtons method. [2]

A3. Find the solution of $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ of $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 18 \\ 2 & 6 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$ using LU Decomposition. [10]

A4. Construct the polynomial interpolating the data using Lagrange polynomials. [10]

x	1	$\frac{1}{2}$	3
y	3	-10	2

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

- B5. (a) Use Newtons method to find all three roots of the function $f(x) = x^3 + 2x^2 - 3x - 1$. You may take an error tolerance of 10^{-4} . [10]
- (b) Use Secants method to find a root of the function $f(x) = x(1 - \cos x)$. You may take an error tolerance of 10^{-4} . [10]
- (c) Solve the system of non-linear algebraic equations

$$x_2 - e^{-x_1} + 1 = 0$$

$$x_1^3 - x_2 = 0$$

using Newtons method (do two iterations). [10]

- B6. (a) Define a cubic spline function fully. [5]
- (b) The data given below is for distance covered by a body at a specified period.

x	0	1	2	4	6
f(x)	1	9	23	93	259

Construct the divided difference table for the data and find $f(4.2)$. [15]

- (c) A river is 80 m wide. The depth of water (in m) of the river at a distance x from one bank is given by the following table:

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find the area of cross-section of the river using trapezoidal rule, [10]

- B7. (a) Use the Gauss Seidel method to solve the system

$$6x_1 - 3x_2 + x_3 = 11,$$

$$2x_1 + x_2 - 8x_3 = -15,$$

$$x_1 - 7x_2 + x_3 = 10.$$

[15]

- (b) Use 4th order Runge Kutta methods to solve $\frac{dy}{dx} = -2y + x + 4$, $y(0) = 1$ and $h = 0.2$. [15]

END OF QUESTION PAPER